Thm 1.1.4⁺ **DeMorgan's Laws**

Let $\{A_i\}_{i \in I}$ be a collection of subsets of $\mathcal{U}$. Let $B \subseteq \mathcal{U}$. Then

$$B \setminus \left(\bigcup_{i \in I} A_i \right) = \bigcap_{i \in I} (B \setminus A_i) \qquad (1)$$

and

$$B \setminus \left(\bigcap_{i \in I} A_i \right) = \bigcup_{i \in I} (B \setminus A_i) \qquad (2)$$

Rmk. When $B = \mathcal{U}$, get

$$\left(\bigcup_{i \in I} A_i \right)^c = \bigcap_{i \in I} (A_i)^c \qquad (1')$$

and

$$\left(\bigcap_{i \in I} A_i \right)^c = \bigcup_{i \in I} (A_i)^c \qquad (2')$$

Pf of Thm 1.1.4⁺ LTGBG.

$\boxed{(1)}$ TFAE <the following are equivalent>

- $x \in B \setminus \left(\bigcup_{i \in I} A_i \right)$

  by def of relative complement
- $x \in B \quad \wedge \quad x \notin \bigcup_{i \in I} A_i$

  by def of union
- $x \in B \quad \wedge \quad \sim [(\exists i \in I)[x \in A_i]]$
- $x \in B \quad \wedge \quad (\forall i \in I)[x \notin A_i]$

  b/c the set B is independent of (has nothing to do with) $i \in I$.
- $(\forall i \in I)[x \in B] \quad \wedge \quad (\forall i \in I)[x \notin A_i]$

  when have 2 $(\forall i \in I)$ connected w/ $\wedge$
- $(\forall i \in I)[x \in B \wedge x \notin A_i]$

  by def of relative complement
- $(\forall i \in I)[x \in B \setminus A_i]$.

  by def of intersection
- $x \in \bigcap_{i \in I} (B \setminus A_i)$.

So (1) holds.

$\boxed{(2)}$ similiar.