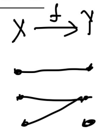
▶. **Set up for this entire handout.** Let $f\colon X \to Y$ be a function from a set X to a set Y. Let $x \in X$ and $y \in Y$. Let $A \subseteq X$ and $B \subseteq Y$. Pictorially have

$$\begin{array}{cccc} f\colon & X & \to & Y \\ & \cup\!| & & \cup\!| \\ & A & & B\,. \end{array}$$



Proposition. Let $A_1, A_2, A_i \subseteq X$ and $B_1, B_2, B_i \subseteq Y$ for each i in an indexing set I. Then

$$f^{-1}\left[\cup_{i\in I}\ B_i\right] \quad = \quad \cup_{i\in I}\ f^{-1}\left[B_i\right] \tag{8}$$

$$f^{-1}\left[\cap_{i\in I}\ B_i\right] \quad = \quad \cap_{i\in I}\ f^{-1}\left[B_i\right]\ . \tag{9}$$

HW : Verify (8).

Below are notes from class for the verification of (9).

(9) Show $f^{-1}\left[\bigcap_{i\in I} B_i\right] = \bigcap_{i\in I} f^{-1}\left[B_i\right]$.

Why TFAE.

- $x \in f^{-1}\left[\bigcap_{i\in I} B_i\right]$

  $\Updownarrow$ def of preimage

- $f(x) \in \bigcap_{i\in I} B_i$

  $\Updownarrow$ def of $\cap$

- $(\forall i \in I)\ \left[f(x) \in B_i\right]$

  $\Updownarrow$ def of preimage

- $(\forall i \in I)\ \left[\ x \in f^{-1}\left[B_i\right]\ \right]$

  $\Updownarrow$ def of $\cap$

- $x \in \bigcap_{i\in I} f^{-1}\left[B_i\right]$