WG1–WG16

WG1. **Know your audience.**

WG2. **Write a first draft of your proof and then revise it.**

WG3. **Keep it simple. Keep it clear.**

WG4. **Use proper grammar.**

WG5. **Use English and minimize the use of cumbersome and unnecessary notation.**

WG6. **Do not use $*$ for multiplication or ^ for exponents. For complicated fractions, do not use /.**

WG7. **Display (by centering) important equations and mathematical expressions.**

WG8. **Equation numbering guidelines.**

WG9. **Watch out for (potentially unclear) words such as: it, this, that.**

WG10. **Separate mathematical symbols/expressions with English words.**

WG11. **Begin each sentence with a word, not a math symbol.**

WG12. **Use the pronouns "we" rather than "I", "you", or "me".**

WG13. **Use italics for variables when using a word processor (i.e., LaTeX).**

WG14. **Keep your reader informed.**

WG15. **Format of Direct Proof of a conditional statement** (i.e., of P (the hypothesis) $\Longrightarrow$ Q (the conclusion)).

WG16. **Format of Other Proofs.**

▷. Our Writing Guidelines (WG) unify the guidelines from the two commonly used Math 300 books:

[S] Mathematical Reasoning. Writing and Proof by Ted Sundstrom, Version 3.
(Appendix A on p. 492-496, also, the guidelines are introduced within sections: 1.1, 1.2, 3.1, 3,3, 4.1.)

[H] *Book of Proof* by Richard Hammack (Edition 3.3, §5.3, p. 133–135)
This book has nice examples of good usage (marked with ✓) and bad usage (marked with ×).

These WG apply not only to mathematics but also to other technical fields (e.g., the STEM fields). Your math writing will evolve with practice. Another way to develop your math and technical writing style is to read other people's writing. Adopt what works and avoid what doesn't.

For further help on a WG, see the right margin. Often given is a reference containing more examples and information. I.e., [S2] refers to the second guideline in [S, Appendix A] while [H3] refers to the 3rd guideline in [H, p. 133–135]. In our below WG, some verbiage is taken directly from [H] and [S]; however, due to space, some items were ommitted. A section (§) number indicates the section in [S] the WG was introduced.

WG1–WG16

WG1. Know your audience. §3.1

Our intended audience is a *confused* fellow classmate in this course. So if in doubt on whether or not to include some verbiage, then ask yourself: "would this verbiage help my confused friend?". [S1]

WG2. Write a first draft of your proof and then revise it. §3.1

Remember that a proof should be written so a reader can easily read and understand the reasoning in the proof. Be clear and concise. First sketch out your proof in your thinking land (i.e., scratch work). Then write your proof in the order of your thinking land implication arrows rather than the order you might have figured out the proof; don't pull the reader through the mud. Include needed details but do not ramble by including unused information. Do not be satisfied with the first draft of a proof. Read it over and refine it. [S15]

WG3. Keep it simple. Keep it clear. §3.1

Use simple declarative sentences and paragraphs, each with a simple point. Order your argument directly (follow the implication arrows in your thinking land), which may differ from the order you produced your thinking land; remember, don't drag your reader through the mud. Do not write out definitions in the book (the reader knows the definitions). Do not include facts not used in your proof. [S14] [H12]

WG4. Use proper grammar. §3.1

Use complete sentences. Avoid run-on sentences. Start sentences with a capital letter. Do not forgot punctuation. Start a new paragraph by indenting (in LaTex, just leave a blank line). [S7] [H2]

WG5. Use English and minimize the use of cumbersome and unnecessary notation. §3.1

In a sentence, do not use: ⟨learn these symbols in Math 300⟩ [S12] [H4] [H5]

$$\forall \ , \ \exists \ , \ \therefore \quad . \qquad \text{(do not use)}$$

In a sentence, do not use logical connectives, such as: ⟨learn these symbols in Math 300⟩

$$\wedge \ , \ \vee \ , \ \Longrightarrow \ , \ \Leftrightarrow \ , \ \sim \ , \ \neg \quad . \qquad \text{(do not use)}$$

In a sentence, you may use standard math notations such as: ⟨knew these symbols before Math 300⟩

$$= \ , \ < \ , \ \leq \ , \ + \ , \ - \ , \ \subseteq \ , \ \sum_{i=1}^{n} \quad . \qquad \text{(can use)}$$

You can use symbols as in (can use) provided the symbols are in a math expression, e.g.," We know $1 + 1 = 2$.". However, the symbol cannot replace the word. E.g., do not say "The numbers x and y are =." but rather say "The numbers x and y are equal." (or just "So we get $x = y$.").

WG6. Do not use $*$ for multiplication or ˆ for exponents. For complicated fractions, do not use /. §3.1

Leave computer science notation for writing computer code, which often is not human-readable friendly. Also avoid using / for division for a complicated fraction. E.g., it is very difficult to read $(x^3 - 3x^2 + 1/2)/(2x/3 - 7)$; however, it is much easier to read the (displayed/centered) fraction [S6]

$$\frac{x^3 - 3x^2 + \frac{1}{2}}{\frac{2x}{3} - 7} \ .$$

WG7. **Display (by centering) important equations and mathematical expressions.** §1.2
Equations and manipulations are often an integral part of a proof. Do not write down an equation that you need to show (even with a question mark above the equal sign) as if it is true before you have shown it is true. Important equations and manipulations should be displayed (i.e., centered) and if one side of an equation does not change, it should not be repeated. [S9]

Example.

Using algebra, we obtain

$$\begin{aligned} x \cdot y &= (2m+1)(2n+1) \\ &= 4mn + 2m + 2n + 1 \\ &= 2(2mn + m + n) + 1. \end{aligned}$$

WG8. **Equation numbering guidelines.** §1.2
If it is necessary to refer to an equation later in a proof, then that equation should be centered and displayed, and it should be given a number. The number for the equation should be written in parentheses on the same line as the equation at the right-hand margin (and is called a label or tag). [S10]

Example Label.
Since x is an odd integer, there exists an integer n such that

$$x = 2n + 1;$$

thus, by algebra

$$x^2 = 2\left(2n^2 + 2n\right) + 1. \tag{1}$$

Since $n \in \mathbb{Z}$, closure properties of $\mathbb{Z}$ gives $2n^2 + 2n \in \mathbb{Z}$. So by (1) and definition of odd, x^2 is odd.
Only number those equations we will be referring to later in the proof.

WG9. **Watch out for (potentially unclear) words such as: it, this, that.** [H9]
The use of such words cause confusion when it is unclear to what the word is referring. If there is any possibility of confusion, you should avoid the word. For example, instead of writing "If $X \subsetneq Y$ then it has strictly more elements." write "If $X \subsetneq Y$ then Y has strictly more elements than X."

WG10. **Separate mathematical symbols/expressions with English words.** §3.1
The reason is clarity. E.g., the sentence "Since $x > 0$, $x^2 > 0$" is hard to read but by just adding a *filler words* such as "Since $x > 0$, we have $x^2 > 0$" makes the sentence easier on the eye. Avoid putting a punctuation symbol between 2 math symbols. Have some go-to *filler words* handy. [S11] [H3]

WG11. **Begin each sentence with a word, not a math symbol.** §3.1
Sentences begin with capital letters, but math symbols are case sensitive. Since n and N have different meanings in math, putting such symbols at the beginning of a sentence leads to ambiguity. [S11] [H1]

WG12. **Use the pronouns "we" rather than "I", "you", or "me".** §1.2
The usual convention in math is to use "we" since it is then as if the reader and writer are having a conversation, with the writer guiding the reader through the details of the proof. [S4] [H6]

WG13. **Use italics for variables when using a word processor (i.e., LaTeX).** §1.2
Of course, most of us cannot italicize when handwriting so just do the best you can. If you are using LaTex, just be sure you are in math mode when typing a variable (e.g., between dollar signs or is a displayed math environment, such as *equation* or *align*). [S5]

WG14. **Keep your reader informed.** §3.3
Make clear and precise what your hypotheses/assumptions are. (E.g.: "Let x and y be odd integers."). §4.1
Explain the meaning of each new symbol introduced. (E.g.: Since x is odd, there is $n \in \mathbb{Z}$ so that $x = 2n + 1$). §1.2
Justify each claim you make. (E.g.: "So xy is odd by definition of odd" or "So xy is odd by Lemma POO."). [S8]

Indicate when the proof is completed. You can put the "end of proof symbol" □ (or ■) at the right-hand margin of the last line of your proof. Another way is to have the one sentence closing paragraph *This completes the proof.* [S13] [H8]

In the beginning of your proof, if you do not indicate your proof method then the reader will think you are using a direct proof. Thus if you are not using a direct proof, then in the beginning of your proof, inform your reader which proof method you are using.

WG15. Format of Direct Proof of a conditional statement (i.e., of P (the hypothesis) $\implies Q$ (the conclusion)). [S2]
Start the 1st line with "Proof." Then begin the 1st (opening) paragraph on the same line by clearly [S3]
stating your hypothesis (or hypotheses, the plural). When stating your hypothesis, you should (natually) [S8]
be setting your notation. Then clearly state what you want to show (WTS) (i.e., the conclusion). [S13]

Next move onto the middle paragraph(s) of your proof, where you argue why your hypothesis implies your WTS. This is the *meat* of your proof. Remember to justify each assertion that is made. Write this part in the order of your thinking land implication arrows rather than the order you might have figured out the proof; remember, don't pull the reader through the mud. Include needed details but do not ramble by including unused information.

Once you have finished arguing that the hypothesis implies the conclusion, inform your reader your proof is finished (with the □ or the sentence *This completes the proof.*).

As an example, consider Exercise 1 (i.e., ER 1). *Show* usually means *prove*.

ER 1. Show that the square of an odd integer is odd.

ER 1 written sybolically is: $(\forall x \in \mathbb{Z})\,[x \text{ is odd} \Rightarrow x^2 \text{ is odd}]$. Below is a (direct) proof of ER 1.

Proof. Let x be an odd integer. We will show that x^2 is odd.

Since x is an odd integer, by definition of odd, there exists an integer n such that

$$x = 2n + 1;$$

thus, by algebra

$$x^2 = 2\left(2n^2 + 2n\right) + 1. \tag{1}$$

Since $n \in \mathbb{Z}$, closure properties of $\mathbb{Z}$ gives $2n^2+2n \in \mathbb{Z}$. So by (1) and definition of odd, x^2 is odd. □

WG16. Format of Other Proofs. [S8]
If not proving a conditional statement by a direct proof, then indicate the proof method at the opening paragraph of your proof. The remainder of the proof varies by method but follows the same guidelines set forth in the above **Format of Direct Proof** and **Keep your reader informed**.

Below are some examples.

If the proof method changes the original statement into a (new and equivalent) conditional statement, proceed as in the format of a direct proof of a conditional statement. See next Theorem.

Theorem 2. The cube root of an irrational real number is irrational.

Thm. 2 symbolically is: $(\forall x \in \mathbb{R})\,[x \text{ is irrational} \Rightarrow \sqrt[3]{x} \text{ is irrational}]$. Below is a proof of Thm. 2.

Proof. We shall show Theorem 2 by contrapositive. Let $x \in \mathbb{R}$ satisfy that $\sqrt[3]{x}$ is rational. We will show that x is rational.

Since $\sqrt[3]{x}$ is rational, there is $q \in \mathbb{Q}$ so that

$$\sqrt[3]{x} = q. \tag{1}$$

Note (1) implies

$$x = q^3. \tag{2}$$

Note $q^3 \in \mathbb{Q}$ since $q \in \mathbb{Q}$ and $\mathbb{Q}$ is closed under multiplication. Thus by (2) we have x is rational. □

If you proof method requires showing several proving parts, then show each part (in separate paragraphs). Remember, keep your reader informed. See next Theorem.

Theorem 3. A real number x is rational if and only if $5x$ is rational.

Thm. 3 symbolically is: $(\forall x \in \mathbb{R})\,[x \in \mathbb{Q} \Leftrightarrow 5x \in \mathbb{Q}]$. Below is a proof of Thm. 3.

Proof. We shall show the biconditional statement in Thm. 3 holds in each direction. Let $x \in \mathbb{R}$.

Towards showing if $x \in \mathbb{Q}$ then $5x \in \mathbb{Q}$, let $x \in \mathbb{Q}$. Since $5 \in \mathbb{Q}$ and $\mathbb{Q}$ is closed under multiplication, $5x \in \mathbb{Q}$.

Towards seeing $5x \in \mathbb{Q}$ implies $x \in \mathbb{Q}$, let $5x \in \mathbb{Q}$. Since $\frac{1}{5} \in \mathbb{Q}$, the rational are closed under multiplication, and $x = \frac{1}{5}(5x)$, we have that $x \in \mathbb{Q}$. □

Other proof methods are handed similarly. In the opening paragraph, state the proof method and, if appropriate, state: hypotheses, WTS, and notation. In the remain paragraph(s), following the WG to show what needs to be shown.