Some definitions/ideas (from number theory) used in the homework exercises.

Def. A natural number p is a **prime number** provided it is greater than 1 and the only natural numbers that are factors of p are: 1 and p. A natural number other than 1 that is not a prime number is a **composite number**. ⟨The number 1 is neither prime nor composite. In Math 546 you will learn 1 is a unit.⟩ p78

Def. An integer n is a **multiple of 3** provided: $(\exists k \in \mathbb{Z})\ [n = 3k]$. p71

Def. A natural number n is a **perfect square** provided: $(\exists k \in \mathbb{N})\ [n = k^2]$. p70

Def. The phrase *for all* (or its equivalents) is a **universal quantifier** and is denoted by $\forall$. p63
The phrase *there exists* (or its equivalents) is an **existential quantifier** and is denoted by $\exists$.

Rmk. The symbol $\exists!$ reads *there exists a unique*. NotInBk

Rmk. Priority/precedence when parentheses are excluded: $\forall$ and $\exists$ and $\exists!$ have equal priority and come after the logical connective symbols: $\sim$ (high) , $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$ (low) . NotInBk

►. Statements with one quantifier. e.g. p64

$$\underbrace{\overbrace{(\,\forall x \in U\,)}^{\text{quantifies the variable } x} \qquad \overbrace{[\,P(x)\,]}^{\text{open sentence in the variable } x}}_{\text{a statement}}$$

Let $P(x)$ be an open sentence of the variable x from the universe U.		
a statement involving	**often has the forms**	**the statement is true provided**
universal quantifier $(\forall x \in U)\ [P(x)]$	For all $x \in U$, $P(x)$. For every $x \in U$, $P(x)$. For each $x \in U$, $P(x)$.	$P(x)$ is true for all $x \in U$.
existential quantifier $(\exists x \in U)\ [P(x)]$	There exists an $x \in U$ such that $P(x)$. There is an $x \in U$ such that $P(x)$.	$P(x)$ is true for at least one $x \in U$.
$(\exists! x \in U)\ [P(x)]$	There exists a unique $x \in U$ such that $P(x)$.	$P(x)$ is true for precisely one (and only one) $x \in U$.

Def. A **counterexample** to a statement of the form $(\forall x \in U)\ [P(x)]$ is an object c in the universal set U for which $P(c)$ is false. So a counterexample is an example that shows $(\forall x \in U)\ [P(x)]$ is false. p69

Ex1a. Do the part a's of Example 1. PT0 ⟨Please Turn Over⟩

Thm. Negations of Quantified Statements. For an open sentence $P(x)$, Thm2.16 p67

$$\sim \{\, (\forall x \in U)\ [P(x)]\,\} \quad \equiv \quad (\exists x \in U)\ [\sim P(x)]$$
$$\sim \{\, (\exists x \in U)\ [P(x)]\,\} \quad \equiv \quad (\forall x \in U)\ [\sim P(x)]$$

Ex1b. Do the part b's of Example 1. PT0 ⟨Please Turn Over⟩

►. Statements with more than one quantifier. p73

	Symbolic Form	**English Form**
Statement	$(\exists x \in \mathbb{Z})\ (\forall y \in \mathbb{Z})\ [x + y = 0]$	There exists an integer x such that for each integer y, we have $x + y = 0$.
Negation	$(\forall x \in \mathbb{Z})\ (\exists y \in \mathbb{Z})\ [x + y \neq 0]$	For each integer x, there exists an integer y such that $x + y \neq 0$.

	Symbolic Form	**English Form**
Statement	$(\forall x \in \mathbb{Z})\ (\exists y \in \mathbb{Z})\ [x + y = 0]$	For each integer x, there is an integer y such that $x + y = 0$.
Negation	$(\exists x \in \mathbb{Z})\ (\forall y \in \mathbb{Z})\ [x + y \neq 0]$	There is an integer x such that for each integer y, we have $x + y \neq 0$.

Ex 2. What about switching order of mixed quantifiers?

Ex 1. **Read** the Symbolically Write Guidelines, which are posted on the course Handout page.
Below are statements from previous Exercises . For each Exercise (ER):

a. Symbolically write (using quantifiers) the original statement.
Then indicate whether the original statement is true or false (no justification needed).

b. Symbolically write (using quantifiers) the negation of the original statement.
Then indicate whether the negation of the original statement is true or false (no justification needed).

1.1. If m is an odd integer, then $5m + 7$ is an even integer. ER1.2.4b p27

1.2. If m and n are odd integers, then $mn + 7$ is an even integer. ER1.2.4c p27
I.e., The sum of 7 and the product of 2 odd integers is an even integer.

1.3. If a, b, and c are integers, then $ab + ac$ is an even integer. ER1.2.7a p28