Def. The phrase *for all* (or its equivalents) is a **universal quantifier** and is denoted by $\forall$. p63
The phrase *there exists* (or its equivalents) is an **existential quantifier** and is denoted by $\exists$.

Rmk. The symbol $\exists!$ reads *there exists a unique.* NotInBk

$\triangleright$. **Statements with one quantifier.** p64

Let $P(x)$ be an open sentence of the variable x from the universe U.		
a statement involving	**often has the forms**	**the statement is true provided**
a universal quantifier: $(\forall x \in U)\ [P(x)]$	For all $x \in U$, $P(x)$. For every $x \in U$, $P(x)$. For each $x \in U$, $P(x)$.	$P(x)$ is true for all $x \in U$.
an existential quantifier: $(\exists x \in U)\ [P(x)]$	There exists an $x \in U$ such that $P(x)$. There is an $x \in U$ such that $P(x)$.	$P(x)$ is true for at least one $x \in U$.
$(\exists! x \in U)\ [P(x)]$	There exists a unique $x \in U$ such that $P(x)$.	$P(x)$ is true for precisely one (and only one) $x \in U$.

Rmk. Priority/precedence when parentheses are excluded: $\forall$ and $\exists$ have equal priority and come after the logical connective symbols: $\sim$ (high) , $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$ (low) . NotInBk

Thm. Negations of Quantified Statements. For an open sentence $P(x)$, Thm2.16 p67

$$\sim \{ (\forall x \in U)\ [P(x)] \} \quad \equiv \quad (\exists x \in U)\ [\sim P(x)]$$

$$\sim \{ (\exists x \in U)\ [P(x)] \} \quad \equiv \quad (\forall x \in U)\ [\sim P(x)]$$

$\triangleright$. A **counterexample** to a statement of the form $(\forall x \in U)\ [P(x)]$ is an object c in the universal set U for which $P(c)$ is false. Hence, a counterexample is an example that proves that $(\forall x \in U)\ [P(x)]$ is a false statement, and hence its negation, $(\exists x \in U)\ [\sim P(x)]$, is a true statement. 69

$\triangleright$. **Statements with more than one quantifier.** p73

	Symbolic Form	**English Form**
Statement	$(\exists x \in \mathbb{Z})\ (\forall y \in \mathbb{Z})\ [x + y = 0]$	There exists an integer x such that for each integer y, we have $x + y = 0$.
Negation	$(\forall x \in \mathbb{Z})\ (\exists y \in \mathbb{Z})\ [x + y \neq 0]$	For each integer x, there exists an integer y such that $x + y \neq 0$.

	Symbolic Form	**English Form**
Statement	$(\forall x \in \mathbb{Z})\ (\exists y \in \mathbb{Z})\ [x + y = 0]$	For each integer x, there is an integer y such that $x + y = 0$.
Negation	$(\exists x \in \mathbb{Z})\ (\forall y \in \mathbb{Z})\ [x + y \neq 0]$	There is an integer x such that for each integer y, we have $x + y \neq 0$.

Ex. Using quantifiers, symbolically write the following previous Exercises. Indicate if the statement is true or false.

1.2.4b. If m is an odd integer, then $5m + 7$ is an even integer. p27

1.2.4c. If m and n are odd integers, then $mn + 7$ is an even integer. p27

1.2.7a. If a, b, and c are integers, then $ab + ac$ is an even integer. p28

1.2.7b. If b,and c are odd integers and a is an integer, then $ab + ac$ is an even integer. p28

Some definitions/ideas (from number theory) used in the homework exercises.

Def. A natural number n is a **perfect square** provided: $(\exists k \in \mathbb{N})\ [n = k^2]$. p70

Def. An integer n is a **multiple of 3** provided: $(\exists k \in \mathbb{Z})\ [n = 3k]$. p71

Def. A natural number p is a **prime number** provided it is greater than 1 and the only natural numbers that are factors of p are: 1 and p. A natural number other than 1 that is not a prime number is a **composite number**. ⟨The number 1 is neither prime nor composite. In Math 546 you will learn 1 is a unit.⟩ p78

Prime Factorization / Fundamental Theorem of Arithmetic

Rmk. Find the prime factoration of 1200.

So $1200 = (12) \cdot (100) = (2 \cdot 3) \cdot (2^2 \cdot 5^2) = 3^1 \cdot 2^3 \cdot 5^2 \underset{\text{better}}{\overset{\text{or}}{=}} 2^3 \cdot 3^1 \cdot 5^2$.
Thus the prime factorization of 1200 is

$$1200 = 2^3 \cdot 3^1 \cdot 5^2. \tag{1}$$

To see what is going on better, note that

(1) there are $m := 3$ distinct primes in the prime factorization of 1200
(2) we found 3 (i.e., m) prime numbers (let's call them: p_1, p_2 and p_3)
(3) we found 3 (i.e., m) natural numbers (let's call them: k_1, k_2 and k_3)

so that

$$1200 = (p_1)^{k_1}\ (p_2)^{k_2}\ (p_3)^{k_3} \tag{2}$$

where

$$p_1 = 2 \quad , \quad p_2 = 3 \quad , \quad p_3 = 5$$
$$k_1 = 3 \quad , \quad k_2 = 1 \quad , \quad k_3 = 2$$

and $p_1 < p_2 < p_3$.

Thm. **Theorem 8.15 (The Fundamental Theorem of Arithmetic)** p432
For each $n \in \mathbb{N} \setminus \{1\}$ there exists unique p427

(1) $m \in \mathbb{N}$
(2) prime numbers $p_1, p_2, \ldots, p_m$
(3) natural numbers $k_1, k_2, \ldots, k_m$

such that

$$n = \prod_{i=1}^{m} (p_i)^{k_i}$$

and $p_1 < p_2 < \ldots < p_{m-1} < p_m$. (We often say: the prime factorization of n is "unique up to ordering")

$\boxed{\text{Recall}}$

When negating quantified statements, we often use the logical equivalencies:

$$[\, \sim (P \Rightarrow Q)\,] \;\equiv\; [\, P \wedge (\sim Q)\,] \qquad \text{(how do you break a promise?)}$$
$$[\, \sim (P \wedge Q)\,] \;\equiv\; [\, P \Rightarrow (\sim Q)\,] \qquad \text{(not in book)}$$
$$[\sim (P \wedge Q)\,] \;\equiv\; [\, (\sim P) \vee (\sim Q)\,] \qquad \text{(De Morgans Law)}$$
$$[\, \sim (P \vee Q)\,] \;\equiv\; [\, (\sim P) \wedge (\sim Q)\,] \qquad \text{(De Morgans Law)}\ .$$

Ex. For the universe of living things, find a negation of *Some kids do not like clowns.*
Solution. Let the universe U be all living things.
Let $K(x)$ be the open sentence "x is a kid".
Let $C(x)$ be the open sentence "x is a clown".
Let $L(x, y)$ be the open sentence "x likes y".
Can you finish?