There will be *proof problems* on the exam. Below are some problems, which you will probably recognize from class, turned into sample (basic, on the easy side) exam *proof problems*. Note the initial parts of a problem often are used in the final part where you actually write your proof. This format is often used with both the easier and harder *proof problems*.

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1. **Theorem 1**. If x and y are odd integers, then xy is an odd integer.

1.1. Write Theorem 1 symbolically. ⟨You can use any appropriate universe(s).⟩ Box answer.

1.2. Complete the definitions (either in English or symbolically).

An integer z is even provided ____ .

An integer z is odd provided ____ .

1.3. On the next 2 pages of lined paper prove Theorem 1 by using the definition of even and odd.

You may not use previously shown results.

If you want you may use the below blank space for Thinking Land.

Optional Thinking Land Space

Finished with proof. Problem 1 Proof is continued on Page ________.

☐ Finished with proof. Problem 1 ☐ Proof is continued on Page ________.

2. **Theorem 2**. If m is an even integer, then

$$3m^2 + 2m + 3$$

is an odd integer.

2.1. Write Theorem 2 symbolically. ⟨You can use any appropriate universe(s).⟩ Box answer.

2.2. On the next pages of lined paper prove Theorem 2 by using the below Previously Shown Results.

Previously Shown Results

Lemma SE1. If m is an even integer, then $m + 1$ is an odd integer.

Lemma SO1. If m is an odd integer, then $m + 1$ is an even integer.

Lemma SEE. If x is an even integer and y is an even integer, then $x + y$ is an even integer.

Lemma SEO. If x is an even integer and y is an odd integer, then $x + y$ is an odd integer.

Lemma SOO. If x is an odd integer and y is an odd integer, then $x + y$ is an even integer.

Lemma PEA. If x is an even integer and y is an integer, then $x \cdot y$ is an even integer.

Lemma POO. If x is an odd integer and y is an odd integer, then $x \cdot y$ is an odd integer.

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3. **Theorem 3**. For each integer a, if 4 divides $(a-1)$ then 4 divides (a^2-1).

Definition 3. A nonzero integer m **divides** an integer n, denoted $m \mid n$, provided that$(\exists q \in \mathbb{Z})\ [n = mq]$.

Remarks.

- Note 5 divides 10, also written $5 \mid 10$, since $10 = 5 \cdot 2$ and $2 \in \mathbb{Z}$.
- The expression "4 divides $(a-1)$" can be written as $4 \mid (a-1)$.
- Note $4 \mid a - 1$ is wrong (and makes absolutely no sense) since $4 \mid a$ is a statement while 1 is a number (you cannot take a statement minus a number).

3.1. Circle True or False (but not both). No justification needed.

TRUE FALSE . $(-5) \mid 10$

TRUE FALSE . $10 \mid 5$

3.2. Write Theorem 3 symbolically. ⟨You can use any appropriate universe(s).⟩ Box answer.

3.3. On the next 2 pages of lined paper prove Theorem 3.

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Optional Thinking Land Space

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