6. **Theorem 6**. For each integer a, if 4 divides $(a-1)$ then 4 divides (a^2-1).

Definition 6. A nonzero integer n **divides** an integer b, denoted $n \mid b$, provided that $(\exists k \in \mathbb{Z})\ [nk = b]$.

Remarks.

- Note 5 divides 10, also written $5 \mid 10$, since $10 = 5 \cdot 2$ and $2 \in \mathbb{Z}$.
- The expression "4 divides $(a-1)$" can be written as $4 \mid (a-1)$.
- Note $4 \mid a - 1$ is wrong (and makes absolutely no sense) since $4 \mid a$ is a statement while 1 is a number (you cannot take a statement minus a number).

6.1. Circle True or False (but not both). No justification needed.

[TRUE]	FALSE .	$(-5) \mid 10$	True since $10 = (-5)(-2)$ and $-2 \in \mathbb{Z}$.
TRUE	[FALSE] .	$10 \mid 5$	

6.2. Write Theorem 6 symbolically. ⟨You can use any appropriate universe(s).⟩ Box answer.

$$\boxed{(\forall a \in \mathbb{Z})\,[\ 4 \mid (a-1) \Rightarrow 4 \mid (a^2-1)\ ]}$$

6.3. On the next 2 pages of lined paper prove Theorem 6.

Proof. Let $a \in \mathbb{Z}$ and $4 \mid (a-1)$. We will show that $4 \mid (a^2-1)$.

Since $4 \mid (a-1)$, by the definition of divides, there is $k \in \mathbb{Z}$ such that

$$4k = a - 1\ . \tag{6.1}$$

Solving equation (6.1) for a gives

$$a = 4k + 1\ . \tag{6.2}$$

Using equation (6.2), followed by algebra, we get

$$\begin{aligned} a^2 - 1 &= (4k+1)^2 - 1 \\ &= 4^2k^2 + 2\,(4k) + 1 - 1 \\ &= 4\left(4k^2 + 2k\right) \\ &= 4j \end{aligned}$$

where $j = 4k^2 + 2k$. Note $j \in \mathbb{Z}$ since $k \in \mathbb{Z}$ and by the closure properties of $\mathbb{Z}$. We have just shown that $a^2 - 1 = 4j$ for some $j \in \mathbb{Z}$. So by definiton of divides we get that $4 \mid (a^2-1)$, which is what we wanted to show.

Consequently, we have shown for each integer a, if $4 \mid (a-1)$ then $4 \mid (a^2-1)$. □