Sample MARK BOX		
PROBLEM	POINTS	
1+2	5+6	
3	10	
4.1–4.2	7	
5.1	5	
6.1–6.2	7	
proof problems		
4.3	20	
5.2	20	
6.3	20	
%	100	

NAME: _____

PIN: _____

INSTRUCTIONS

- The MARK BOX above indicates the problems (check that you have them all) along with their points.
- Write your PIN on the top center on **each** page.
- **Write on only the front side of the paper**. Only the front side will graded. Use pencil and easer (no scratchouts).
- On this exam you may use any prior result on this exam, even if you could not show it (similar to with homework).
- When writing a proof, follow the proof *Writing Guidelines*. When writing symbolically, follow the *Symbolically Write Guidelines* (don't forget needed quantifiers). We have been using these guidelines in class.
- A *proof problem*'s first page might have basic questions (space provided for your answer), follow by *optional* Thinking Land space towards the bottom.
  The problem's first page is then followed by some pages of lined paper.
  Write your proof on the provided lined paper. No need to skip lines.
- Upon request, you will be given as much (lined and/or blank-scratch) paper as you need.
- If you run out of space for a solution at the place the solution should go, then leave me a note at the where you solution should be telling me where to go look for the rest of your solution.
- During the exam, the use of unauthorized materials is prohibited. Unauthorized materials include: books, notes, eletronic devices (e.g., cell phone, smart watch, earbuds) Unauthorized materials (including cell phones) must be in a secured (e.g. zipped up, snapped closed) bag placed completely under your desk or, if you did not bring such a bag, give it to Prof. Girardi to hold for you during the exam (and it will be returned when you leave the exam). This means no electronic devices allowed in your pockets. At a student's request, I will project a clock to the screen.
- During this exam, do not leave your seat unless you have permission. If you have a question, raise your hand.
- When you finish: turn your exam over, put your pencil down and raise your hand.
- Not following instructions can result in a lose of points. Cheating is grounds for a F in the course.
- This exam covers (from *Mathematical Reasoning* by Sundstrom, Version 3): §1.1–2.5.

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.

As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

I understand that if it is determined that I used any unauthorized assistance or otherwise violated the University's Honor Code then I will receive a failing grade for this course and be referred to the academic Dean and the Office of Academic Integrity for additional disciplinary actions.

Furthermore, I have not only read but will also follow the instructions on the exam.

Signature : _____

1–3. The first few problems will be mostly fill in the blank/box, multiple choice, and true/false problems to make sure you know the basic definitions and have comprehended the basic ideas.

4–6. The next few problems will be *proof problems*.

The next pages contain some sample proof problems (many of which you will might recognize from class) turned into sample (basic, on the easy side) exam *proof problems*.

Some items to notice about proof problems.

(1) The initial parts of a problem often are used in the final part where you actually write your proof. This format is often used with both the easier and harder proof problems.

(2) You need to know how to symbolically write a statement (do not forget to quantify).

(3) If the proof uses definitions from the sections covered on this test, then you need to know the definition.

(4) If the proof uses new definitions, then you will be given the definition.

(5) The Thinking Lands are optional.

4. The purpose of this problem is to prove Lemma POO from class.

Theorem 4. If x and y are odd integers, then xy is an odd integer.

4.1. Symbolically write Theorem 4. Box your answer. ⟨Don't forget your quantifiers.⟩

4.2. Complete the definitions symbolically (not in English).

An integer z is even provided _____ .

An integer z is odd provided _____ .

4.3. On the next pages of lined paper, carefully prove Theorem 4 by using the definition of even/odd integers. You may not use *Previously Shown Results* (such as Lemma POO). Below is space for your optional Thinking Land.

Optional Thinking Land Space

☐ Finished with proof. Problem 4 ☐ Proof is continued on Page _____.

☐ Finished with proof. Problem 4 ☐ Proof is continued on Page _____.

5. **Theorem 5**. If m is an even integer, then

$$3m^2 + 2m + 3$$

is an odd integer.

5.1. Symbolically write Theorem 5. Box your answer. Do not forget your quantifier(s).

5.2. On the next pages of lined paper, carefully prove Theorem 5 by using the below listed previously shown results. You may use the parity ⟨even/odd-ness⟩ of specific integers (e.g.: 3 is odd, 2 is even).

Previously Shown Results.

Lemma SEE. The sum of two even integers is an even integer.

Lemma SEO. The sum of an even integer and an odd integer is an odd integer.

Lemma SOO. The sum of two odd integers is an even integer.

Lemma PEA. The product of an even integer and any integer is an even integer.

Lemma POO. The product of two odd integers is an odd integer.

Optional Thinking Land Space

☐ Finished with proof. Problem 5 ☐ Proof is continued on Page _____.

☐ Finished with proof. Problem 5 ☐ Proof is continued on Page _____.

6. **Theorem 6**. For each integer a, if 4 divides $(a-1)$ then 4 divides (a^2-1).

Definition 6. A nonzero integer n **divides** an integer b, denoted $n \mid b$, provided that $(\exists k \in \mathbb{Z})\ [nk = b]$.

Remarks.

- Note 5 divides 10, also written $5 \mid 10$, since $10 = 5 \cdot 2$ and $2 \in \mathbb{Z}$.
- The expression "4 divides $(a-1)$" can be written as $4 \mid (a-1)$.
- Note $4 \mid a-1$ is wrong (and makes absolutely no sense) since $4 \mid a$ is a statement while 1 is a number (you cannot take a statement minus a number).

6.1. Circle True or False (but not both). No justification needed.

TRUE FALSE . $(-5) \mid 10$

TRUE FALSE . $10 \mid 5$

6.2. Symbolically write Theorem 6. Box your answer. Do not forget your quantifier(s).

6.3. On the next pages of lined paper, carefully prove Theorem 6.

Optional Thinking Land Space

☐ Finished with proof. Problem 6 ☐ Proof is continued on Page _____.

☐ Finished with proof. Problem 6 ☐ Proof is continued on Page _____.

Extra Scratch Paper (in case you need it).
The professor will not look at (nor grade) anything on this page.
If you want, you can rip this page off and not hand it in.