Practice Problems from Section 3.6. Exercises 3.6.1–3.6.13 except for: 2, 10, 13.
These *Practice Problems* are a sampling of the type of problems which could be on the exam.
These *Practice Problem* are, in no way, meant as a comprehensive review for the exam.

Math is not a spectator sport.
Often we learn more from our failed attempts at a proof rather than
immediately looking at the hints or reading a clean proof.
So give these problems a solid attempt before seeking help, e.g.:
looking through: your notes, the texbook, the homework.
Since these problems are not to hand in, you may share:
hints and/or an attempt at a solution for others to comment on.

Some hints are very generous. Do not except such generous hints on the exam.

1. **Theorem 1**. If $n \in \mathbb{N}$ with $n \geq 2$, then

$$\prod_{i=2}^{n} \frac{i^2-1}{i^2} = \frac{n+1}{2n} .$$

1.1. Symbolically write Theorem 1. (Do not forget your quantifiers.)

1.2. Prove Theorem 1 using math induction..

∘. Recall, the sum $\sum_{i=2}^{5} a_i = a_2 + a_3 + a_4 + a_5$ while the product $\prod_{i=2}^{5} a_i = a_2 \cdot a_3 \cdot a_4 \cdot a_5$

hint. The sum $\sum_{i=2}^{n+1} a_i = (a_2 + a_3 + \ldots + a_n) + a_{n+1} = \left(\sum_{i=2}^{n} \right) + a_{n+1}$. This idea is often used in the inductive step. Similarly, for the product $\prod_{i=2}^{n+1} a_i = (a_2 \cdot a_3 \cdot \ \cdots \ \cdot a_n) \cdot a_{n+1} = \left(\prod_{i=2}^{n} a_i \right) \cdot a_{n+1}$.

2. **Theorem 2**. If $n \in \mathbb{N}$ with $n \geq 3$, then

$$2n + 1 < 2^n .$$

2.1. Symbolically write Theorem 2. (Do not forget your quantifiers.)

2.2. Prove Theorem 2 using math induction.

3. **Theorem 3**. If $n \in \mathbb{Z}$ with $n \geq 0$, then

$$3 \mid \left(n^3 + 2n \right) .$$

3.1. Symbolically write Theorem 3. (Do not forget your quantifiers.)

3.2. Prove Theorem 3 using math induction.

hint. See book Proposition 4.4 (p178) and (starred) ER 3.4.8a (p181).

4. **Theorem 4**. If $n \in \mathbb{Z}$ with $n \geq 0$, then

$$8 \mid (9^n - 1) .$$

4.1. Symbolically write Theorem 4. (Do not forget your quantifiers.)

4.2. Prove Theorem 4 using math induction.

hint. See book Proposition 4.4 (p178) and (starred) ER 3.4.8a (p181).

5. **Theorem 5**. Let $\{x_n\}_{n=1}^{\infty}$ be the recursively defined sequence defined by

$$x_1 = 1 \quad , \quad x_2 = 2$$

and

$$\text{when } n \in \mathbb{N}, \qquad x_{n+2} = \frac{x_{n+1} + x_n}{2} . \tag{RD}$$

If $n \in \mathbb{N}$, then

$$1 \leq x_n \leq 2 .$$

5.1. Symbolically write Theorem 5. (Do not forget your quantifiers.)

5.2. Prove Theorem 5 using math induction.

hint. Note you need to prove both the lower bound and upper bound.

6. **Theorem 6**. Let $\{x_n\}_{n=1}^{\infty}$ be the recursively defined sequence be

$$x_1 = 1 \quad , \quad x_2 = 1 \quad , \quad x_3 = 1$$

and

$$\text{when } n \in \mathbb{N}^{\geq 4}, \qquad x_n = x_{n-1} + x_{n-2} + x_{n-3} . \tag{RD}$$

If $n \in \mathbb{N}$, then

$$x_n < 2^n .$$

6.1. Symbolically write Theorem 6. (Do not forget your quantifiers.)

6.2. Prove Theorem 6 using math induction.