Hints on MML HW 12.4 GeoGebra Question :

Find vectors $\vec{u}$ and $\vec{v}$ so that $\vec{u} \times \vec{v} = \vec{w}$

The vector $\vec{w}$ varies, let's say in your variant, $\vec{w} = \langle 2, 4, 6 \rangle$.

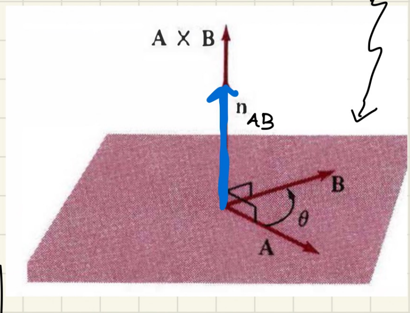
This question is generating requests for hints so let's give the whole class a hint.

Recall these notes from class :

- Def

$\vec{n}_{AB}$ is the right-hand-rule unit vector perpendicular ($\perp$) to the plane $\mathcal{P}_{AB}$.

$\mathcal{P}_{AB}$



see picture

- Since $\vec{n}_{AB}$ is a unit vector, $\boxed{\|\vec{n}_{AB}\| = 1}$.
- Since $\vec{n}_{AB} \perp \mathcal{P}_{AB}$, we get $\boxed{\vec{n}_{AB} \perp \vec{A}}$ and $\boxed{\vec{n}_{AB} \perp \vec{B}}$
- Right-hand-rule (RHR) stuff for $\vec{n}_{AB}$.

Def. Cross Product

note scalar is positive.

$$\underbrace{\vec{A} \times \vec{B}}_{\text{vectors}} = \underbrace{\overbrace{\left[\|\vec{A}\| \, \|\vec{B}\| \sin\theta_{AB}\right]}^{\text{scalar}} \overbrace{\vec{n}_{AB}}^{\text{vector}}}_{\text{a vector}} \qquad (1)$$

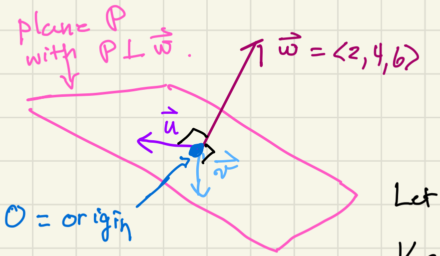
Important take-away : $\boxed{(\vec{A} \times \vec{B}) \perp \vec{A}}$ and $\boxed{(\vec{A} \times \vec{B}) \perp \vec{B}}$.

We also learned in class that if $\vec{A} = \langle x_A, y_A, z_A \rangle$ and $\vec{B} = \langle x_B, y_B, z_B \rangle$ then

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_A & y_A & z_A \\ x_B & y_B & z_B \end{vmatrix}$$

Now back to our example:

Find vectors $\vec{u}$ and $\vec{v}$ so that $\vec{u} \times \vec{v} = \vec{w}$ where $\vec{w} = \langle 2,4,6 \rangle$.



Let

$\vec{u} = \langle u_1, u_2, u_3 \rangle$ and

$\vec{v} = \langle v_1, v_2, v_3 \rangle$

Let $\mathscr{P}$ be the plane $\perp$ to $\vec{w}$ and thru the $\overbrace{\text{origin } (0,0,0)}^{O \,\equiv}$.

Key Fix the tails of $\vec{u}, \vec{v}, \vec{w}$ to be the origin.

Then $\vec{u} \times \vec{v}$ is on the line containing $\vec{w}$

so $\vec{u} \times \vec{v} = k\vec{w}$ for some scalar k

so $\vec{u} \times \frac{\vec{v}}{k} = \vec{w}$.

Recall $\vec{u} \times \vec{v} = \vec{w}$

From previous page, we know:

do we want • or X ?   do we want 0 or $\vec{0}$?

(1) $(\vec{u} \times \vec{v}) \perp \vec{u}$. Know $\vec{w} \perp \vec{u} \iff \langle 2,4,6 \rangle \boxed{} \langle u_1, u_2, u_3 \rangle = \boxed{}$

(2) $(\vec{u} \times \vec{v}) \perp \vec{v}$. Know $\vec{w} \perp \vec{v} \iff \langle 2,4,6 \rangle \boxed{} \langle v_1, v_2, v_3 \rangle = \boxed{}$

(3) $\vec{w} = \vec{u} \times \vec{v} \iff \langle 2,4,6 \rangle = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} \iff \langle 2,4,6 \rangle = \langle ?, ?, ? \rangle$

Now, (1) – (3) will give a system of equations, which will have many solutions (look at picture to see this).

To start, find an "easy" $\vec{u}$ satisfying (1), i.e. $\vec{w} \perp \vec{u}$.

One such "easy" $\vec{u}$ has the form $\langle \underline{?}, \underline{?}, 0 \rangle$.

Then, using this "easy" $\vec{u}$, find $\vec{v}$ so that (2) and (3) hold.

(i.e. find v_1, v_2, v_3)