At $x = u$, $\boxed{}$	has the <u>indeterminate form</u> $\boxed{}$	if $\lim_{x \to u} f(x) = \boxed{}$	and $\lim_{x \to u} g(x) = \boxed{}$
(1) $\frac{f(x)}{g(x)}$	$\frac{0}{0}$	0	0
(2) $\frac{f(x)}{g(x)}$	$\frac{\infty}{\infty}$	∞	∞
(3) $f(x) \cdot g(x)$	$0 \cdot \infty$	0	∞
(4) $f(x) - g(x)$	$\infty - \infty$	∞	∞
(5) $[f(x)]^{g(x)}$	0^0	0	0
(6) $[f(x)]^{g(x)}$	∞^0	∞	0
(7) $[f(x)]^{g(x)}$	1^∞	1	∞

HERE: u stands for any of the symbols a, a^-, a^+, $-\infty$, $+\infty$.

L'Hôpital's Rule

(1) and (2)

If:

- $\frac{f(x)}{g(x)}$ has the indeterminate form $\boxed{\frac{0}{0}}$ or $\boxed{\frac{\infty}{\infty}}$ at u

and

- $\lim\limits_{x \to u} \dfrac{f'(x)}{g'(x)}$ exists (i.e. this limit is a finite number or $-\infty$ or ∞)

then

$$\lim_{x \to u} \frac{f(x)}{g(x)} = \lim_{x \to u} \frac{f'(x)}{g'(x)} .$$

(3)

If $f(x) \cdot g(x)$ has the interdeterminate form $\boxed{0 \cdot \infty}$ at u, then rewrite:

$$f(x) \cdot g(x) = \frac{f(x)}{1/g(x)}$$, which has the interdeterminate form $\boxed{\frac{0}{0}}$ at u

or

$$f(x) \cdot g(x) = \frac{g(x)}{1/f(x)}$$, which has the interdeterminate form $\boxed{\frac{\infty}{\infty}}$ at u

and then apply L'Hôpital's Rule.

(4)

If $f(x) - g(x)$ has the interdeterminate form $\boxed{\infty - \infty}$ at u,

then use algebraic manipulation to convert $f(x) - g(x)$

into a form of the type $\boxed{\frac{0}{0}}$ or $\boxed{\frac{\infty}{\infty}}$

and then apply L'Hôpital's Rule.

(5)

If $[f(x)]^{g(x)}$ has the interdeterminate form $\boxed{0^0}$ at u, then follow these steps:

Let

$$y = [f(x)]^{g(x)} \ .$$

So

$$\ln y = \ln \Big(\ [\ f(x) \ ]^{g(x)} \ \Big) \ .$$

Next, simplify

$$\ln y = [g(x)] \ \cdot \ \ln [f(x)] \ .$$

Note that $\ln y = [g(x)] \ \cdot \ \ln [f(x)]$ has the interdeterminate form $\boxed{0 \ \cdot \ -\infty}$ at u.

Using an appropriate above method (i.e. **(3)**), evaluate

$$\lim_{x \to u} \ \ln y \ \equiv \ L \ .$$

Conclude

$$\lim_{x \to u} \ \ln [f(x)]^{g(x)} \ = \ L \qquad \Longrightarrow \qquad \lim_{x \to u} [f(x)]^{g(x)} \ = \ e^L \ .$$

(6) and **(7)**

If $[f(x)]^{g(x)}$ has the interdeterminate form $\boxed{\infty^0}$ or $\boxed{1^\infty}$, then proceed similarly as in **(5)** .

Note that $\ln y = [g(x)] \ \cdot \ \ln [f(x)]$ will have the interdeterminate form

(6) $\boxed{0 \ \cdot \ \infty}$ at u

(7) $\boxed{\infty \ \cdot \ 0}$ at u .