Helpful Overleaf Feature. If you left double click at a place in the PDF file, then Overleaf indicates the corresponding place in the LaTeX file, making it easy to compare the PDF output to LaTex input.

How to write a proof.

Basically, we start our proof with \begin{proof} and end our proof with \end{proof}. Then just put our proof between the \begin{proof} and \end{proof}. That's all. Recall in LaTex as soon as we write \begin{proof}, in order to compile we need to include the \end{proof} below the \begin{proof}, In LaTex, whenever we *begin* something, we need to *end* that something before compiling. The below example is a slight modification to the proof of Theorem 1.8 on page 22.

Lemma POO. The product of two odd integers is an odd integer.

Proof. Let x and y be odd integers. We will show that $x \cdot y$ is an odd integer.

Since x and y are odd integers, by definition of odd integer, there exist $k_x, k_y \in \mathbb{Z}$ such that

$$x = 2k_x + 1 \quad \text{and} \quad y = 2k_y + 1. \tag{1}$$

By (1) and then algebra,

$$\begin{aligned} x \cdot y &= (2k_x + 1)(2k_y + 1) \\ &= 4k_x k_y + 2k_x + 2k_y + 1 \\ &= 2(2k_x k_y + k_x + k_y) + 1 \\ &= 2q + 1 \end{aligned} \tag{2}$$

where $q = 2k_x k_y + k_x + k_y$. Note $q \in \mathbb{Z}$ since $2, k_x, k_y \in \mathbb{Z}$ and the integers are closed under multiplication and addition. We have just shown

$$x \cdot y = 2q + 1 \text{ for some } q \in \mathbb{Z}. \tag{3}$$

Thus equation (3) shows that $x \cdot y$ is an odd integer by the definition of odd integer.

We have just shown that the product of two odd integers is an odd integer. □

BTW. Below is a Know-Show Table for this theorem.

Know-Show Table			
optional columns		**mandatory columns**	
order	step	**Know**	**Reason**
1	P	x and y are odd integers	Hypothesis
4	P1	There exists integers k_x and k_y such that $x = 2k_x + 1$ and $y = 2k_y + 1$	definition of odd integer
5	P2	$xy = (2k_x + 1)(2k_y + 1)$	substitution
6	P3	$xy = 4k_x k_y + 2k_x + 2k_y + 1$	algebra
7	P4	$xy = 2(2k_x k_y + k_x + k_y) + 1$	algebra
8	P5	$(2k_x k_y + k_x + k_y)$ is an integer	closure properties of the integers
3	Q1	There exists an integer q such that $xy = 2q + 1$.	use $q = (2k_x k_y + k_x + k_y)$
2	Q	$x \cdot y$ is an odd integer.	definition of odd integer

Remember, the orderand step columns may vary.