High School Math Contest

University of South Carolina

January 19, 2002

Problem 4:

Problem 4:

Average for women: 83
Average for men: 71
Average for all: 80

What percentage of the students are women?

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Average for women: 83 Number of women: w

Average for men: 71

Average for all: 80

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Average for men: 71
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$=$ sum of women scores $+$ sum of men scores

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$$\begin{aligned} 80 \cdot N &= \text{sum of the scores in the class} \\ &= \text{ sum of women scores } + \text{ sum of men scores} \end{aligned}$$

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$$\begin{aligned} \mathbf{80} \cdot \boldsymbol{N} &= \text{sum of the scores in the class} \\ &= \underbrace{\text{sum of women scores}}_{\mathbf{83} \cdot \boldsymbol{w}} + \underbrace{\text{sum of men scores}}_{\mathbf{71} \cdot (\boldsymbol{N} - \boldsymbol{w})} \end{aligned}$$

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$$\boldsymbol{80N = 83w + 71(N - w)}$$

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$$\frac{\boldsymbol{w}}{\boldsymbol{N}}$$

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$$\frac{w}{N} = \frac{9}{12}$$

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$$\frac{w}{N} = \frac{9}{12} = 75\%$$

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$$\frac{w}{N} = \frac{9}{12} = \boxed{75\%}$$

Problem 5:

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$$3^a = 4, \quad 4^b = 5, \quad 5^c = 6$$
$$6^d = 7, \quad 7^e = 8, \quad 8^f = 9$$

Calculate $abcdef$?

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$$3^{abcdef}$$

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$$3^{abcdef} = (\ 4\ )^{bcdef}$$

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Calculate $abcdef$?

$$3^{abcdef} = (\,4\,)^{bcdef}$$
$$= (4^b)^{cdef}$$

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$$6^d = 7, \quad 7^e = 8, \quad 8^f = 9$$

Calculate $abcdef$?

$$\begin{aligned} 3^{abcdef} &= (\,4\,)^{bcdef} \\ &= (4^b)^{cdef} \end{aligned}$$

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Calculate $abcdef$?

$$\begin{aligned} 3^{abcdef} &= (\,4\,)^{bcdef} \\ &= (\,5\,)^{cdef} \\ &= \ \ 6^{def} \end{aligned}$$

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$$3^a = 4, \quad 4^b = 5, \quad 5^c = 6$$
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Calculate $abcdef$?

$$\begin{aligned} 3^{abcdef} &= (\ 4\ )^{bcdef} \\ &= (\ 5\ )^{cdef} \\ &= \quad 7^{ef} \end{aligned}$$

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Calculate $abcdef$?

$$\begin{aligned} 3^{abcdef} &= (\ 4\ )^{bcdef} \\ &= (\ 5\ )^{cdef} \\ &= \ \ 8^{f} \end{aligned}$$

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Calculate $abcdef$?

$$\begin{aligned} 3^{abcdef} &= (\ 4\ )^{bcdef} \\ &= (\ 5\ )^{cdef} \\ &= \ \ 9 \end{aligned}$$

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$$\mathbf{3}^{a} = \mathbf{4}, \quad \mathbf{4}^{b} = \mathbf{5}, \quad \mathbf{5}^{c} = \mathbf{6}$$

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Calculate $abcdef$?

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Calculate $abcdef$?

$$\begin{aligned} 3^{abcdef} &= \left(4 \right)^{bcdef} \\ &= \left(5 \right)^{cdef} \\ &= \ \ 9 \end{aligned}$$

$$abcdef = 2$$

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Calculate $abcdef$?

$$\begin{aligned} 3^{abcdef} &= (\,4\,)^{bcdef} \\ &= (\,5\,)^{cdef} \\ &= \;\; 9 \end{aligned}$$

$$abcdef = \boxed{2}$$

Problem 7:

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30 multiple choice questions

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5 points for a correct answer

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Which of the scores 147, 144, 143, 141, 139 is possible?

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Which of the scores 147, 144, 143, 141, 139 is possible?

x problems correct, y with no answer

$\implies$ $0 \leq x + y \leq 30$ and the score is $5x + y$

PROBLEM 7:

For each $S \in \{147, 144, 143, 141, 139\}$, are there non-negative integers x and y such that

$$0 \leq x + y \leq 30 \quad \text{and} \quad S = 5x + y\,?$$

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$$141 = 5 \times 28 + 1$$

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$$141 = 5 \times 28 + 1 \implies \text{the answer is } 141$$

PROBLEM 7:

For each $\boldsymbol{S} \in \{\mathbf{147}, \mathbf{144}, \mathbf{143}, \mathbf{141}, \mathbf{139}\}$, are there non-negative integers $\boldsymbol{x}$ and $\boldsymbol{y}$ such that

$$\mathbf{0} \leq \boldsymbol{x} + \boldsymbol{y} \leq \mathbf{30} \quad \text{and} \quad \boldsymbol{S} = \mathbf{5}\boldsymbol{x} + \boldsymbol{y}\,?$$

$$\mathbf{141} = \mathbf{5} \times \mathbf{28} + \mathbf{1} \implies \text{the answer is } \mathbf{141}$$

Is this the only correct answer?

Problem 7:

For each $\boldsymbol{S} \in \{\mathbf{147}, \mathbf{144}, \mathbf{143}, \mathbf{141}, \mathbf{139}\}$, are there non-negative integers $\boldsymbol{x}$ and $\boldsymbol{y}$ such that

$$\mathbf{0} \leq \boldsymbol{x} + \boldsymbol{y} \leq \mathbf{30} \quad \text{and} \quad \boldsymbol{S} = \mathbf{5}\boldsymbol{x} + \boldsymbol{y}\,?$$

$$\boldsymbol{S} = \mathbf{5}\boldsymbol{x} + \boldsymbol{y}$$

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For each $S \in \{147, 144, 143, 141, 139\}$, are there non-negative integers x and y such that

$$0 \leq x + y \leq 30 \quad \text{and} \quad S = 5x + y\,?$$

$$S = 5x + y = 5(x + y) - 4y$$

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$$S = 5x + y = 5(x + y) - 4y \leq 150 - 4y$$

$$S = 5x + y$$

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$$S = 5x + y = 5(x + y) - 4y \leq 150 - 4y$$

$$S = 5x + y \leq 150 - 4 \cdot 2 = 142$$

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For each $\boldsymbol{S \in \{147, 144, 143, 141, 139\}}$, are there non-negative integers $\boldsymbol{x}$ and $\boldsymbol{y}$ such that

$$\boldsymbol{0 \leq x + y \leq 30} \quad \text{and} \quad \boldsymbol{S = 5x + y}\,?$$

$$\boldsymbol{S = 5x + y = 5(x + y) - 4y \leq 150 - 4y}$$

$$\boldsymbol{S = 5x + y}$$

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For each $S \in \{147, 144, 143, 141, 139\}$, are there non-negative integers x and y such that

$$0 \leq x + y \leq 30 \quad \text{and} \quad S = 5x + y\,?$$

$$S = 5x + y = 5(x + y) - 4y \leq 150 - 4y$$

$$S = 5x + y \leq 150 - 4 \cdot 3 = 138$$

What’s the smallest score that
is not obtainable on this test?

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The only nonobtainable scores are:

139

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$\mathbf{139, 143}$

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$$\mathbf{139, 143, 144}$$

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$$\mathbf{139, 143, 144, 147}$$

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The only nonobtainable scores are:

$$\mathbf{139, 143, 144, 147, 148, 149}$$

What's the smallest score that is not obtainable on this test?

The only nonobtainable scores are:

$$\mathbf{139, 143, 144, 147, 148, 149}$$

and the obvious ones like:

$$\mathbf{151, -28, \sqrt{2}, \sqrt{-1}, \frac{\boldsymbol{\pi}^2}{\boldsymbol{e} - 17}}$$

Problem 10:

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$$\frac{a+b}{c} = \frac{b+c}{a} = \frac{c+a}{b}$$

Problem 10:

$$\frac{a+b}{c} = \frac{b+c}{a} = \frac{c+a}{b} = ?$$

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a, b, and c are different

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Main Idea:

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$$\frac{a+b}{c} = \frac{b+c}{a} = \frac{c+a}{b} = ?$$

a, b, and c are different

Main Idea: Add **1** to each expression.

Problem 10:

$$\frac{a+b}{c} = \frac{b+c}{a} = \frac{c+a}{b} = ?$$

Main Idea: Add 1 to each expression.

Problem 10:

$$\frac{a+b}{c} = \frac{b+c}{a} = \frac{c+a}{b} = ?$$

Main Idea: Add 1 to each expression.

$$1 + \frac{a+b}{c}$$

Problem 10:

$$\frac{a+b}{c} = \frac{b+c}{a} = \frac{c+a}{b} = ?$$

Main Idea: Add 1 to each expression.

$$1 + \frac{a+b}{c} = \frac{a+b+c}{c}$$

Problem 10:

$$\frac{\boldsymbol{a}+\boldsymbol{b}}{\boldsymbol{c}} = \frac{\boldsymbol{b}+\boldsymbol{c}}{\boldsymbol{a}} = \frac{\boldsymbol{c}+\boldsymbol{a}}{\boldsymbol{b}} = ?$$

Main Idea: Add 1 to each expression.

$$\mathbf{1} + \frac{\boldsymbol{a}+\boldsymbol{b}}{\boldsymbol{c}} = \frac{\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c}}{\boldsymbol{c}}$$

$$\mathbf{1} + \frac{\boldsymbol{b}+\boldsymbol{c}}{\boldsymbol{a}}$$

Problem 10:

$$\frac{\boldsymbol{a+b}}{\boldsymbol{c}} = \frac{\boldsymbol{b+c}}{\boldsymbol{a}} = \frac{\boldsymbol{c+a}}{\boldsymbol{b}} = ?$$

Main Idea: Add **1** to each expression.

$$\mathbf{1} + \frac{\boldsymbol{a+b}}{\boldsymbol{c}} = \frac{\boldsymbol{a+b+c}}{\boldsymbol{c}}$$

$$\mathbf{1} + \frac{\boldsymbol{b+c}}{\boldsymbol{a}} = \frac{\boldsymbol{a+b+c}}{\boldsymbol{a}}$$

Problem 10:

$$\frac{a+b}{c} = \frac{b+c}{a} = \frac{c+a}{b} = ?$$

Main Idea: Add 1 to each expression.

$$1 + \frac{a+b}{c} = \frac{a+b+c}{c}$$

$$1 + \frac{b+c}{a} = \frac{a+b+c}{a}$$

$$1 + \frac{c+a}{b}$$

Problem 10:

$$\frac{a+b}{c} = \frac{b+c}{a} = \frac{c+a}{b} = ?$$

Main Idea: Add 1 to each expression.

$$1 + \frac{a+b}{c} = \frac{a+b+c}{c}$$

$$1 + \frac{b+c}{a} = \frac{a+b+c}{a}$$

$$1 + \frac{c+a}{b} = \frac{a+b+c}{b}$$

Problem 10:

$$\frac{a+b}{c} = \frac{b+c}{a} = \frac{c+a}{b} = ?$$

$$\frac{a+b+c}{a} = \frac{a+b+c}{b} = \frac{a+b+c}{c}$$

Problem 10:

$$\frac{a+b}{c} = \frac{b+c}{a} = \frac{c+a}{b} = ?$$

$$\frac{1}{a} \qquad = \qquad \frac{1}{b} \qquad = \qquad \frac{1}{c}$$

Problem 10:

$$\frac{a+b}{c} = \frac{b+c}{a} = \frac{c+a}{b} = ?$$

$$\frac{1}{a} \qquad = \qquad \frac{1}{b} \qquad = \qquad \frac{1}{c}$$

$$a = b = c$$

Problem 10:

$$\frac{a+b}{c} = \frac{b+c}{a} = \frac{c+a}{b} = ?$$

$$\frac{a+b+c}{a} = \frac{a+b+c}{b} = \frac{a+b+c}{c}$$

Problem 10:

$$\frac{a+b}{c} = \frac{b+c}{a} = \frac{c+a}{b} = ?$$

$$\frac{a+b+c}{a} = \frac{a+b+c}{b} = \frac{a+b+c}{c}$$

$$a+b+c=0$$

Problem 10:

$$\frac{a+b}{c}=\frac{b+c}{a}=\frac{c+a}{b}=?$$

$$\frac{a+b+c}{a}=\frac{a+b+c}{b}=\frac{a+b+c}{c}$$

$$a+b+c=0$$

$$\frac{a+b}{c}$$

Problem 10:

$$\frac{a+b}{c} = \frac{b+c}{a} = \frac{c+a}{b} = ?$$

$$\frac{a+b+c}{a} = \frac{a+b+c}{b} = \frac{a+b+c}{c}$$

$$a+b+c=0$$

$$\frac{a+b}{c} = -1$$

Problem 10:

$$\frac{a+b}{c} = \frac{b+c}{a} = \frac{c+a}{b} = ?$$

$$\frac{a+b+c}{a} = \frac{a+b+c}{b} = \frac{a+b+c}{c}$$

$$a+b+c=0$$

$$\frac{a+b}{c} = \boxed{-1}$$

Problem 10:

$$\frac{a+b}{c} = \frac{b+c}{a} = \frac{c+a}{b} = ?$$

Take $a = 1$, $b = 2$, and $c = -3$.

Problem 18:

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$$a + b + c + d = 10 \qquad (a+b)(c+d) = 16$$

$$(a+c)(b+d) = 21 \qquad (a+d)(b+c) = 24$$

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$$(a+c)(b+d)=21 \qquad (a+d)(b+c)=24$$

$$a^2+b^2+c^2+d^2=?$$

Problem 18:

$$a+b+c+d=10 \qquad (a+b)(c+d)=16$$
$$(a+c)(b+d)=21 \qquad (a+d)(b+c)=24$$
$$a^2+b^2+c^2+d^2=?$$

$$\begin{aligned} a^2+b^2+c^2+d^2 \\ &= (a+b+c+d)^2-(a+b)(c+d) \\ &\qquad -(a+c)(b+d)-(a+d)(b+c) \end{aligned}$$

Problem 18:

$$a+b+c+d=10 \qquad (a+b)(c+d)=16$$
$$(a+c)(b+d)=21 \qquad (a+d)(b+c)=24$$
$$a^2+b^2+c^2+d^2=?$$

$$\begin{aligned} a^2+b^2+c^2+d^2 &\\ &= (a+b+c+d)^2-(a+b)(c+d)\\ &\qquad -(a+c)(b+d)-(a+d)(b+c)\\ &= 10^2-16-21-24=39 \end{aligned}$$

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$$a+b+c+d=10 \qquad (a+b)(c+d)=16$$
$$(a+c)(b+d)=21 \qquad (a+d)(b+c)=24$$
$$a^2+b^2+c^2+d^2=?$$

$$\begin{aligned}
a^2+b^2+c^2+d^2 \\
&= (a+b+c+d)^2-(a+b)(c+d) \\
&\qquad -(a+c)(b+d)-(a+d)(b+c) \\
&= 10^2-16-21-24=\boxed{39}
\end{aligned}$$

Problem 18:

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\begin{aligned}
a^2+b^2+c^2+d^2 &= (a+b+c+d)^2-(a+b)(c+d) \\
&\qquad -(a+c)(b+d)-(a+d)(b+c)
\end{aligned}
$$

Problem 18:

$$\begin{aligned} a^2+b^2+c^2+d^2 &= (a+b+c+d)^2 - (a+b)(c+d) \\ &\qquad - (a+c)(b+d) - (a+d)(b+c) \end{aligned}$$

$$\begin{aligned} (a+b+c+d)^2 &= a^2+b^2+c^2+d^2+(a+b)(c+d) \\ &\qquad + (a+c)(b+d) + (a+d)(b+c) \end{aligned}$$

Problem 18:

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\begin{aligned}
a^2+b^2+c^2+d^2 \\
&= (a+b+c+d)^2 - (a+b)(c+d) \\
&\quad - (a+c)(b+d) - (a+d)(b+c)
\end{aligned}
$$

$$
\begin{aligned}
(a+b+c+d)(a+b+c+d) \\
&= a^2+b^2+c^2+d^2+(a+b)(c+d) \\
&\quad + (a+c)(b+d) + (a+d)(b+c)
\end{aligned}
$$

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\begin{aligned}
a^2+b^2+c^2+d^2 &= (a+b+c+d)^2-(a+b)(c+d)\\
&\quad -(a+c)(b+d)-(a+d)(b+c)
\end{aligned}
$$

$$
\begin{aligned}
(a+b+c+d)(a+b+c+d) &= a^2+b^2+c^2+d^2+(a+b)(c+d)\\
&\quad +(a+c)(b+d)+(a+d)(b+c)
\end{aligned}
$$

Problem 18:

$$\begin{aligned} a^2+b^2+c^2+d^2 &= (a+b+c+d)^2-(a+b)(c+d) \\ &\quad -(a+c)(b+d)-(a+d)(b+c) \end{aligned}$$

$$\begin{aligned} (a+b+c+d)(a+b+c+d) &= a^2+b^2+c^2+d^2+(a+b)(c+d) \\ &\quad +(a+c)(b+d)+(a+d)(b+c) \end{aligned}$$

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$$\begin{aligned} a^2+b^2+c^2+d^2 &= (a+b+c+d)^2-(a+b)(c+d) \\ &\quad -(a+c)(b+d)-(a+d)(b+c) \end{aligned}$$

$$\begin{aligned} (a+b+c+d)(a+b+c+d) &= a^2+b^2+c^2+d^2+(a+b)(c+d) \\ &\quad +(a+c)(b+d)+(a+d)(b+c) \end{aligned}$$

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\begin{aligned}
a^2+b^2+c^2+d^2 \\
&= (a+b+c+d)^2 - (a+b)(c+d) \\
&\qquad - (a+c)(b+d) - (a+d)(b+c)
\end{aligned}
$$

$$
\begin{aligned}
(a+{\color{red}b}+c+d)(a+{\color{red}b}+c+d) \\
&= a^2+{\color{red}b^2}+c^2+d^2+(a+b)(c+d) \\
&\qquad + (a+c)(b+d) + (a+d)(b+c)
\end{aligned}
$$

Problem 18:

$$\begin{aligned} a^2+b^2+c^2+d^2 &= (a+b+c+d)^2-(a+b)(c+d) \\ &\qquad -(a+c)(b+d)-(a+d)(b+c) \end{aligned}$$

$$\begin{aligned} (a+b+c+d)(a+b+c+d) &= a^2+b^2+c^2+d^2+(a+b)(c+d) \\ &\qquad +(a+c)(b+d)+(a+d)(b+c) \end{aligned}$$

Problem 18:

$$a^2+b^2+c^2+d^2$$
$$= (a+b+c+d)^2 - (a+b)(c+d)$$
$$- (a+c)(b+d) - (a+d)(b+c)$$

$$(a+b+c+d)(a+b+c+d)$$
$$= a^2+b^2+c^2+d^2+(a+b)(c+d)$$
$$+ (a+c)(b+d) + (a+d)(b+c)$$

Problem 18:

$$
\begin{aligned}
a^2+b^2+c^2+d^2 \\
&= (a+b+c+d)^2 - (a+b)(c+d) \\
&\qquad - (a+c)(b+d) - (a+d)(b+c)
\end{aligned}
$$

$$
\begin{aligned}
(\color{red}{a} + b + c + d)(a + \color{red}{b} + c + d) \\
&= a^2 + b^2 + c^2 + d^2 + (a+b)(c+d) \\
&\qquad + (a+c)(b+d) + (a+d)(b+c)
\end{aligned}
$$

Problem 18:

$$
\begin{aligned}
a^2+b^2+c^2+d^2 \\
&= (a+b+c+d)^2 - (a+b)(c+d) \\
&\quad - (a+c)(b+d) - (a+d)(b+c)
\end{aligned}
$$

$$
\begin{aligned}
&(\color{red}{a}\color{blue}{} + \color{green}{b} + c + d)(\color{green}{a} + \color{red}{b} + c + d) \\
&\qquad = a^2 + b^2 + c^2 + d^2 + (a+b)(c+d) \\
&\qquad\quad + (a+c)(b+d) + (a+d)(b+c)
\end{aligned}
$$

Problem 18:

$$\begin{aligned} a^2+b^2+c^2+d^2 &= (a+b+c+d)^2-(a+b)(c+d) \\ &\quad -(a+c)(b+d)-(a+d)(b+c) \end{aligned}$$

$$\begin{aligned} (a+b+c+d)(a+b+c+d) &= a^2+b^2+c^2+d^2+(a+b)(c+d) \\ &\quad +(a+c)(b+d)+(a+d)(b+c) \end{aligned}$$

Problem 18:

$$
\begin{aligned}
a^2+b^2+c^2+d^2 &= (a+b+c+d)^2-(a+b)(c+d) \\
&\quad -(a+c)(b+d)-(a+d)(b+c)
\end{aligned}
$$

$$
\begin{aligned}
(a+b+c+d)(a+b+c+d) &= a^2+b^2+c^2+d^2+(a+b)(c+d) \\
&\quad +(a+c)(b+d)+(a+d)(b+c)
\end{aligned}
$$

Problem 18:

$$
\begin{aligned}
a^2+b^2+c^2+d^2 \\
&= (a+b+c+d)^2 - (a+b)(c+d) \\
&\qquad - (a+c)(b+d) - (a+d)(b+c)
\end{aligned}
$$

$$
\begin{aligned}
(a+b+c+d)(a+b+c+d) \\
&= a^2+b^2+c^2+d^2+(a+b)(c+d) \\
&\qquad + (a+c)(b+d) + (a+d)(b+c)
\end{aligned}
$$

Problem 18:

$$a+b+c+d=10 \qquad (a+b)(c+d)=16$$

$$(a+c)(b+d)=21 \qquad (a+d)(b+c)=24$$

$$a^2+b^2+c^2+d^2=?$$

Problem 18:

$$a+b+c+d=10 \qquad (a+b)(c+d)=16$$

$$(a+c)(b+d)=21 \qquad (a+d)(b+c)=24$$

$$a^2+b^2+c^2+d^2=\,?$$

$$\big(x-(a+b)\big)\big(x-(c+d)\big)$$

Problem 18:

$$a+b+c+d=10 \qquad (a+b)(c+d)=16$$
$$(a+c)(b+d)=21 \qquad (a+d)(b+c)=24$$
$$a^2+b^2+c^2+d^2=?$$

$$\big(x-(a+b)\big)\big(x-(c+d)\big)$$
$$=x^2-(a+b+c+d)x+(a+b)(c+d)$$

Problem 18:

$$a+b+c+d=10 \qquad (a+b)(c+d)=16$$
$$(a+c)(b+d)=21 \qquad (a+d)(b+c)=24$$
$$a^2+b^2+c^2+d^2=\ ?$$

$$\big(x-(a+b)\big)\big(x-(c+d)\big)$$
$$=x^2-(a+b+c+d)x+(a+b)(c+d)$$
$$=x^2-10x+16$$

Problem 18:

$$a+b+c+d=10 \qquad (a+b)(c+d)=16$$
$$(a+c)(b+d)=21 \qquad (a+d)(b+c)=24$$
$$a^2+b^2+c^2+d^2=?$$

$$\begin{aligned}\big(x-(a+b)\big)\big(x-(c+d)\big) \\ &= x^2-(a+b+c+d)x+(a+b)(c+d) \\ &= x^2-10x+16 \\ &= (x-2)(x-8)\end{aligned}$$

Problem 18:

$$a+b+c+d=10 \qquad a+b=2$$
$$(a+c)(b+d)=21 \qquad (a+d)(b+c)=24$$
$$a^2+b^2+c^2+d^2=?$$

$$\begin{aligned}\big(x-(a+b)\big)\big(x-(c+d)\big) \\ &= x^2-(a+b+c+d)x+(a+b)(c+d) \\ &= x^2-10x+16 \\ &= (x-2)(x-8)\end{aligned}$$

Problem 18:

$$\boldsymbol{a+b+c+d=10} \qquad \boldsymbol{a+b=2}$$
$$\boldsymbol{(a+c)(b+d)=21} \qquad \boldsymbol{(a+d)(b+c)=24}$$
$$\boldsymbol{a^2+b^2+c^2+d^2=?}$$

$$\boldsymbol{\big(x-(a+c)\big)\big(x-(b+d)\big)}$$
$$\boldsymbol{=x^2-(a+b+c+d)x+(a+c)(b+d)}$$
$$\boldsymbol{=x^2-10x+21}$$
$$\boldsymbol{=(x-3)(x-7)}$$

Problem 18:

$$a+b+c+d=10 \qquad a+b=2$$
$$a+c=3 \qquad (a+d)(b+c)=24$$
$$a^2+b^2+c^2+d^2=?$$

$$\begin{aligned}\big(x-(a+c)\big)\big(x-(b+d)\big)\\ &= x^2-(a+b+c+d)x+(a+c)(b+d)\\ &= x^2-10x+21\\ &= (x-3)(x-7)\end{aligned}$$

Problem 18:

$$a+b+c+d=10 \qquad a+b=2$$
$$a+c=3 \qquad (a+d)(b+c)=24$$
$$a^2+b^2+c^2+d^2=?$$

$$\begin{aligned}
&\big(x-(a+d)\big)\big(x-(b+c)\big)\\
&\qquad = x^2-(a+b+c+d)x+(a+d)(b+c)\\
&\qquad = x^2-10x+24\\
&\qquad = (x-4)(x-6)
\end{aligned}$$

Problem 18:

$$a+b+c+d=10 \qquad a+b=2$$

$$a+c=3 \qquad a+d=4$$

$$a^2+b^2+c^2+d^2=?$$

$$\begin{aligned}
&\big(x-(a+d)\big)\big(x-(b+c)\big)\\
&\qquad = x^2-(a+b+c+d)x+(a+d)(b+c)\\
&\qquad = x^2-10x+24\\
&\qquad = (x-4)(x-6)
\end{aligned}$$

Problem 18:

$$a + b + c + d = 10 \qquad a + b = 2$$

$$a + c = 3 \qquad a + d = 4$$

$$a^2 + b^2 + c^2 + d^2 = ?$$

Problem 18:

$$a+b+c+d=10 \qquad a+b=2$$

$$a+c=3 \qquad a+d=4$$

$$a^2+b^2+c^2+d^2=?$$

$$\begin{aligned}2a &= (a+b)+(a+c)+(a+d)\\ &\qquad -(a+b+c+d)\\ &= -1\end{aligned}$$

Problem 18:

$$a + b + c + d = 10 \qquad a + b = 2$$

$$a + c = 3 \qquad a + d = 4$$

$$a^2 + b^2 + c^2 + d^2 = ?$$

$$\begin{aligned} 2a &= (a+b) + (a+c) + (a+d) \\ &\qquad - (a+b+c+d) \\ &= -1 \end{aligned}$$

$$a = -1/2$$

Problem 18:

$$a + b + c + d = 10 \qquad a + b = 2$$

$$a + c = 3 \qquad a + d = 4$$

$$a^2 + b^2 + c^2 + d^2 = ?$$

$$\begin{aligned} 2a &= (a + b) + (a + c) + (a + d) \\ &\qquad - (a + b + c + d) \\ &= -1 \end{aligned}$$

$$a = -1/2, \quad b = 5/2$$

Problem 18:

$$a+b+c+d=10 \qquad a+b=2$$
$$a+c=3 \qquad a+d=4$$
$$a^2+b^2+c^2+d^2=?$$

$$\begin{aligned} 2a &= (a+b)+(a+c)+(a+d) \\ &\qquad -(a+b+c+d) \\ &= -1 \end{aligned}$$

$$a=-1/2, \quad b=5/2, \quad c=7/2, \quad d=9/2$$

Problem 19:

Problem 19:

$$[x] = \text{ greatest integer } \leq x$$

Problem 19:

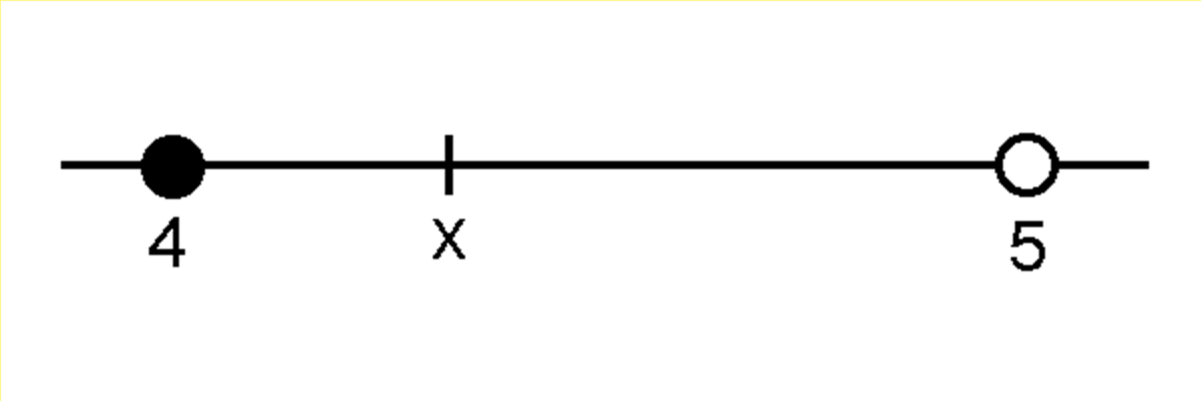
$$\lfloor x \rfloor = \text{ greatest integer } \leq x$$

Problem 19:

$$[x] = \text{ greatest integer } \leq x$$

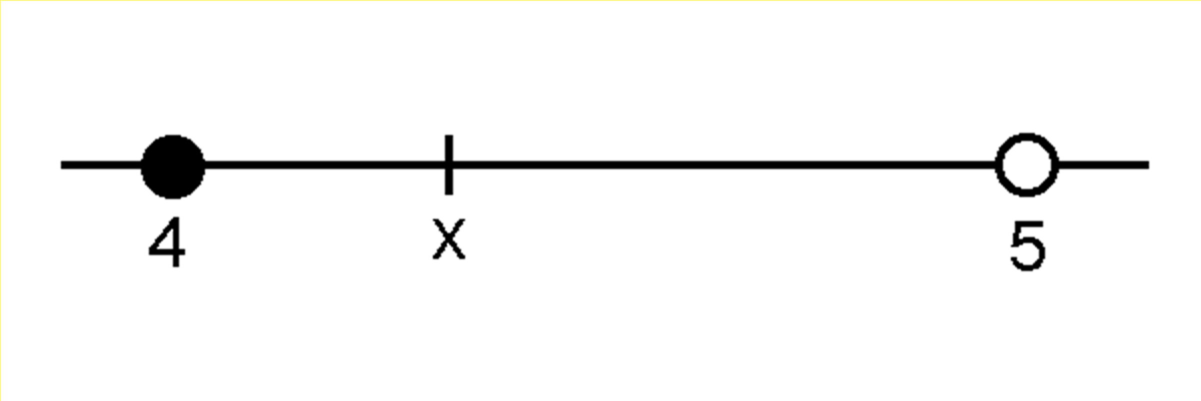
Problem 19:

$$[x] = \text{ greatest integer } \leq x$$



Problem 19:

$$[x] \;=\; \text{greatest integer } \;\leq x$$



$$4 \leq x < 5 \;\Longrightarrow\; [x] = 4$$

Problem 19:

$$a \cdot [a] = 17, \quad b \cdot [b] = 11, \quad a - b = \,?$$

Problem 19:

$$\boldsymbol{a \cdot [a] = 17, \quad b \cdot [b] = 11, \quad a - b = ?}$$

$$\boldsymbol{4 \leq a}$$

Problem 19:

$$a \cdot [a] = 17, \quad b \cdot [b] = 11, \quad a - b = ?$$

$$4 \leq a < 5$$

Problem 19:

$$a \cdot [a] = 17, \quad b \cdot [b] = 11, \quad a - b = ?$$

$$4 \leq a < 5 \implies [a] = 4$$

Problem 19:

$$a \cdot [a] = 17, \quad b \cdot [b] = 11, \quad a - b = ?$$

$$4 \le a < 5 \implies [a] = 4$$

Problem 19:

$$\boldsymbol{a \cdot [a] = 17, \quad b \cdot [b] = 11, \quad a - b = ?}$$

$$\boldsymbol{4 \le a < 5 \implies [a] = 4 \implies a = \frac{17}{4}}$$

Problem 19:

$$a \cdot [a] = 17, \quad b \cdot [b] = 11, \quad a - b = ?$$

$$4 \leq a < 5 \implies [a] = 4 \implies a = \frac{17}{4}$$

$$3 \leq b < 4$$

Problem 19:

$$\boldsymbol{a \cdot [a] = 17, \quad b \cdot [b] = 11, \quad a - b = ?}$$

$$\boldsymbol{4 \le a < 5 \implies [a] = 4 \implies a = \frac{17}{4}}$$

$$\boldsymbol{3 \le b < 4 \implies [b] = 3}$$

Problem 19:

$$a \cdot [a] = 17, \quad b \cdot [b] = 11, \quad a - b = ?$$

$$4 \leq a < 5 \implies [a] = 4 \implies a = \frac{17}{4}$$

$$3 \leq b < 4 \implies [b] = 3 \implies b = \frac{11}{3}$$

Problem 19:

$$a \cdot [a] = 17, \quad b \cdot [b] = 11, \quad a - b = ?$$

$$4 \le a < 5 \implies [a] = 4 \implies a = \frac{17}{4}$$

$$3 \le b < 4 \implies [b] = 3 \implies b = \frac{11}{3}$$

$$a - b = \frac{17}{4} - \frac{11}{3}$$

Problem 19:

$$a \cdot [a] = 17, \quad b \cdot [b] = 11, \quad a - b = ?$$

$$4 \le a < 5 \implies [a] = 4 \implies a = \frac{17}{4}$$

$$3 \le b < 4 \implies [b] = 3 \implies b = \frac{11}{3}$$

$$a - b = \frac{17}{4} - \frac{11}{3} = \frac{51 - 44}{12}$$

Problem 19:

$$a \cdot [a] = 17, \quad b \cdot [b] = 11, \quad a - b = ?$$

$$4 \leq a < 5 \implies [a] = 4 \implies a = \frac{17}{4}$$

$$3 \leq b < 4 \implies [b] = 3 \implies b = \frac{11}{3}$$

$$a - b = \frac{17}{4} - \frac{11}{3} = \frac{7}{12}$$

Problem 19:

$$a \cdot [a] = 17, \quad b \cdot [b] = 11, \quad a - b = ?$$

$$4 \leq a < 5 \implies [a] = 4 \implies a = \frac{17}{4}$$

$$3 \leq b < 4 \implies [b] = 3 \implies b = \frac{11}{3}$$

$$a - b = \frac{17}{4} - \frac{11}{3} = \boxed{\frac{7}{12}}$$

Problem 22:

Problem 22:

$$1000 \cdot 1000! + 999 \cdot 999! + \cdots + 2 \cdot 2! + 1 \cdot 1! = \,?$$

Problem 22:

$$1000 \cdot 1000! + 999 \cdot 999! + \cdots + 2 \cdot 2! + 1 \cdot 1! = \ ?$$

Main Idea:

Problem 22:

$$1000 \cdot 1000! + 999 \cdot 999! + \cdots + 2 \cdot 2! + 1 \cdot 1! = ?$$

Main Idea: $(n+1)! = (n+1) \cdot n!$

Problem 22:

$$\mathbf{1000 \cdot 1000! + 999 \cdot 999! + \cdots + 2 \cdot 2! + 1 \cdot 1! = ?}$$

Main Idea: $(n+1)! = (n+1) \cdot n! = n \cdot n! + n!$

Problem 22:

$$1000 \cdot 1000! + 999 \cdot 999! + \cdots + 2 \cdot 2! + 1 \cdot 1! = ?$$

Main Idea: $(n+1)! = (n+1) \cdot n! = n \cdot n! + n!$

Problem 22:

$$1000 \cdot 1000! + 999 \cdot 999! + \cdots + 2 \cdot 2! + 1 \cdot 1! = ?$$

Main Idea: $(n+1)! = n \cdot n! + n!$

Problem 22:

$$1000 \cdot 1000! + 999 \cdot 999! + \cdots + 2 \cdot 2! + 1 \cdot 1! = ?$$

Main Idea: $(n+1)! = n \cdot n! + n!$

$$1001! = 1000 \cdot 1000! + 1000!$$

Problem 22:

$$1000 \cdot 1000! + 999 \cdot 999! + \cdots + 2 \cdot 2! + 1 \cdot 1! = ?$$

Main Idea: $(n+1)! = n \cdot n! + n!$

$$\begin{aligned} 1001! &= 1000 \cdot 1000! + 1000! \\ &= 1000 \cdot 1000! + 999 \cdot 999! + 999! \end{aligned}$$

Problem 22:

$$1000 \cdot 1000! + 999 \cdot 999! + \cdots + 2 \cdot 2! + 1 \cdot 1! = ?$$

Main Idea: $(n+1)! = n \cdot n! + n!$

$$\begin{aligned} 1001! &= 1000 \cdot 1000! + 1000! \\ &= 1000 \cdot 1000! + 999 \cdot 999! + 999! \\ &= 1000 \cdot 1000! + 999 \cdot 999! + 998 \cdot 998! + 998! \end{aligned}$$

Problem 22:

$$\mathbf{1000 \cdot 1000! + 999 \cdot 999! + \cdots + 2 \cdot 2! + 1 \cdot 1! = ?}$$

Main Idea: $(n+1)! = n \cdot n! + n!$

$$\begin{aligned} 1001! &= 1000 \cdot 1000! + 1000! \\ &= 1000 \cdot 1000! + 999 \cdot 999! + 999! \\ &= 1000 \cdot 1000! + 999 \cdot 999! + 998 \cdot 998! + 998! \\ &= \cdots = 1000 \cdot 1000! + \cdots + 2 \cdot 2! + 1 \cdot 1! + 1! \end{aligned}$$

Problem 22:

$$\mathbf{1000 \cdot 1000! + 999 \cdot 999! + \cdots + 2 \cdot 2! + 1 \cdot 1! = ?}$$

Main Idea: $\boldsymbol{(n+1)! = n \cdot n! + n!}$

$$\begin{aligned}
\mathbf{1001!} &= \mathbf{1000 \cdot 1000! + 1000!} \\
&= \mathbf{1000 \cdot 1000! + 999 \cdot 999! + 999!} \\
&= \mathbf{1000 \cdot 1000! + 999 \cdot 999! + 998 \cdot 998! + 998!} \\
&= \cdots = \underbrace{\mathbf{1000 \cdot 1000! + \cdots + 2 \cdot 2! + 1 \cdot 1!}}_{?} \mathbf{+1!}
\end{aligned}$$

Problem 22:

$$\mathbf{1000 \cdot 1000! + \cdots + 2 \cdot 2! + 1 \cdot 1! = 1001! - 1}$$

Problem 22:

$$\begin{aligned} 1000 \cdot 1000! + \cdots + 2 \cdot 2! + 1 \cdot 1! &= 1001! - 1 \\ &= 2002 \cdot k - 1 \end{aligned}$$

Problem 22:

$$
\begin{aligned}
1000 \cdot 1000! + \cdots + 2 \cdot 2! + 1 \cdot 1! &= 1001! - 1 \\
&= 2002 \cdot k - 1 \\
&= 2002(k-1) + 2001
\end{aligned}
$$

Problem 22:

$$\begin{aligned} 1000 \cdot 1000! + \cdots + 2 \cdot 2! + 1 \cdot 1! &= 1001! - 1 \\ &= 2002 \cdot k - 1 \\ &= 2002(k-1) + 2001 \end{aligned}$$

Problem 22:

$$
\begin{aligned}
1000 \cdot 1000! + \cdots + 2 \cdot 2! + 1 \cdot 1! &= 1001! - 1 \\
&= 2002 \cdot k - 1 \\
&= 2002(k-1) + \underbrace{\boxed{2001}}_{\uparrow\ \text{answer}}
\end{aligned}
$$

Problem 23:

Problem 23:

$$j = \text{ John's age when Tammy was 4}$$
$$m = \text{ Martha's age when Tammy was 4}$$

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$$j = \text{ John's age when Tammy was 4}$$
$$m = \text{ Martha's age when Tammy was 4}$$

$$j = 3m,$$

Problem 23:

$$j = \text{ John's age when Tammy was 4}$$
$$m = \text{ Martha's age when Tammy was 4}$$

$$\boldsymbol{j = 3m, \quad m + a = 2(4 + a),}$$

Problem 23:

$$j = \text{ John's age when Tammy was 4}$$
$$m = \text{ Martha's age when Tammy was 4}$$

$$j = 3m, \quad m + a = 2(4 + a),$$
$$j + a = 5(4 + a),$$

Problem 23:

$$j = \text{ John's age when Tammy was 4}$$
$$m = \text{ Martha's age when Tammy was 4}$$

$$\boldsymbol{j = 3m, \quad m + a = 2(4 + a),}$$
$$\boldsymbol{j + a = 5(4 + a), \quad j + b = 2(m + b)}$$

Problem 23:

$$j = \text{ John's age when Tammy was 4}$$

$$m = \text{ Martha's age when Tammy was 4}$$

$$\boldsymbol{j = 3m, \quad m + a = 2(4 + a),}$$

$$\boldsymbol{j + a = 5(4 + a), \quad j + b = 2(m + b)}$$

$$\boldsymbol{4 + b =} \ ?$$

Problem 23:

$$j = 3m, \quad m + a = 2(4 + a),$$
$$j + a = 5(4 + a), \quad j + b = 2(m + b)$$

$$4 + b = \ ?$$

Problem 23:

$$j = 3m, \quad m + a = 2(4 + a),$$
$$j + a = 5(4 + a), \quad j + b = 2(m + b)$$

$$4 + b = \ ?$$

$$b =$$

Problem 23:

$$j = 3m, \quad m + a = 2(4 + a),$$
$$j + a = 5(4 + a), \quad j + b = 2(m + b)$$

$$4 + b = \ ?$$

$$b = j - 2m =$$

Problem 23:

$$j = 3m, \quad m + a = 2(4 + a),$$
$$j + a = 5(4 + a), \quad j + b = 2(m + b)$$

$$4 + b = \ ?$$

$$b = j - 2m = 3m - 2m = m$$

Problem 23:

$$j = 3m, \quad m + a = 2(4 + a),$$
$$j + a = 5(4 + a), \quad j + b = 2(m + b)$$

$$4 + b = \ ?$$

$$b = j - 2m = 3m - 2m = m$$

$$m =$$

Problem 23:

$$j = 3m, \quad m + a = 2(4 + a),$$
$$j + a = 5(4 + a), \quad j + b = 2(m + b)$$

$$4 + b = \ ?$$

$$b = j - 2m = 3m - 2m = m$$
$$m = 8 + a,$$

Problem 23:

$$j = 3m, \quad m + a = 2(4 + a),$$
$$j + a = 5(4 + a), \quad j + b = 2(m + b)$$

$$4 + b = \ ?$$

$$b = j - 2m = 3m - 2m = m$$
$$m = 8 + a, \quad 3m =$$

Problem 23:

$$j = 3m, \quad m + a = 2(4 + a),$$
$$j + a = 5(4 + a), \quad j + b = 2(m + b)$$

$$4 + b = \ ?$$

$$b = j - 2m = 3m - 2m = m$$
$$m = 8 + a, \quad 3m = 20 + 4a$$

Problem 23:

$$j = 3m, \quad m + a = 2(4 + a),$$
$$j + a = 5(4 + a), \quad j + b = 2(m + b)$$

$$4 + b = \ ?$$

$$b = j - 2m = 3m - 2m = m$$
$$m = 8 + a, \quad 3m = 20 + 4a$$

$b = m$

Problem 23:

$$j = 3m, \quad m + a = 2(4 + a),$$
$$j + a = 5(4 + a), \quad j + b = 2(m + b)$$

$$4 + b = \ ?$$

$$b = j - 2m = 3m - 2m = m$$
$$m = 8 + a, \quad 3m = 20 + 4a$$

$b = m = 4m - 3m =$

Problem 23:

$$j = 3m, \quad m + a = 2(4 + a),$$
$$j + a = 5(4 + a), \quad j + b = 2(m + b)$$

$$4 + b = \ ?$$

$$b = j - 2m = 3m - 2m = m$$
$$m = 8 + a, \quad 3m = 20 + 4a$$
$$b = m = 4m - 3m = 4(8 + a) - (20 + 4a)$$

Problem 23:

$$j = 3m, \quad m + a = 2(4 + a),$$
$$j + a = 5(4 + a), \quad j + b = 2(m + b)$$

$$4 + b = \ ?$$

$$b = j - 2m = 3m - 2m = m$$
$$m = 8 + a, \quad 3m = 20 + 4a$$
$$b = m = 4m - 3m = 4(8 + a) - (20 + 4a) = 12$$

Problem 23:

$$j = 3m, \quad m + a = 2(4 + a),$$
$$j + a = 5(4 + a), \quad j + b = 2(m + b)$$

$$4 + b = \boxed{16}$$

$$b = j - 2m = 3m - 2m = m$$
$$m = 8 + a, \quad 3m = 20 + 4a$$
$$b = m = 4m - 3m = 4(8 + a) - (20 + 4a) = 12$$

Remarks on Writing a High School Competition:

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Original Problem 23: When Tammy was four years old, John was three times as old as Martha. When Martha was twice as old as Tammy, John was five times as old as Tammy. How old was Tammy when John was twice as old as Martha?

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Concern: What does it mean to say someone is some age?

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Concern: What does it mean to say someone is some age? When someone is 8.75 years old, don't we say they are 8?

Remarks on Writing a High School Competition:

Original Problem 23: When Tammy was four years old, John was three times as old as Martha. When Martha was twice as old as Tammy, John was five times as old as Tammy. How old was Tammy when John was twice as old as Martha?

Concern: What does it mean to say someone is some age? When someone is 8.75 years old, don’t we say they are 8? Does that really make a difference in the problem?

Remarks on Writing a High School Competition:

Original Problem 23: When Tammy was four years old, John was three times as old as Martha. When Martha was twice as old as Tammy, John was five times as old as Tammy. How old was Tammy when John was twice as old as Martha? (Recall Answer: 16)

Remarks on Writing a High School Competition:

Original Problem 23: When Tammy was four years old, John was three times as old as Martha. When Martha was twice as old as Tammy, John was five times as old as Tammy. How old was Tammy when John was twice as old as Martha? (Recall Answer: 16)

	Tammy	John	Martha
	4	24	8 (really **8.75**)

Remarks on Writing a High School Competition:

Original Problem 23: When Tammy was four years old, John was three times as old as Martha. When Martha was twice as old as Tammy, John was five times as old as Tammy. How old was Tammy when John was twice as old as Martha? (Recall Answer: 16)

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1.5	5	25	10

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6.5	12	32	16

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Remarks on Writing a High School Competition:

Revised Problem (Draft 5): Two days ago, Bobby had 4 marbles and Greg had three times as many marbles as Peter. Yesterday, their mother gave each of them some additional marbles (each boy received the same amount), so that Peter had twice as many marbles as Bobby, and Greg had five times as many marbles as Bobby. Today, their father gave each of them some additional marbles (each boy received the same amount), so that Greg now has twice as many marbles as Peter. How many marbles does Bobby have today?

Remarks on Writing a High School Competition:

Yet Another Version: Let f_1, f_2, and f_3 be the functions $x + a$, $x + b$, and $x + c$ in some order where a, b, and c are real numbers. Suppose there are real numbers x_1, x_2, and x_3 satisfying:

(i) $f_1(x_1) = 4$ and $f_3(x_1) = 3f_2(x_1)$

(ii) $f_2(x_2) = 2f_1(x_2)$ and $f_3(x_2) = 5f_1(x_2)$

(iii) $f_3(x_3) = 2f_2(x_3)$

What is the value of $f_1(x_3)$?

Remarks on Writing a High School Competition:

The Best Idea (my opinion):

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Use a different problem!!

Remarks on Writing a High School Competition:

Actual Problem 23: Tammy, John, and Martha were all born at noon on January 19th, but in different years. When Tammy was four years old, John was three times as old as Martha. When Martha was twice as old as Tammy, John was five times as old as Tammy. How old was Tammy when John was twice as old as Martha?

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$$\mathbf{2}\boldsymbol{n}+\mathbf{3} \text{ divides } \mathbf{5}\boldsymbol{n}+\mathbf{26}$$

$$\Downarrow$$

$$\mathbf{2}\boldsymbol{n}+\mathbf{3} \text{ divides } \mathbf{2}(\mathbf{5}\boldsymbol{n}+\mathbf{26})-\mathbf{5}(\mathbf{2}\boldsymbol{n}+\mathbf{3})=\mathbf{37}$$

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For how many integers n is this the case?

$$2n+3 \text{ divides } 5n+26$$

$$\Updownarrow$$

$$2n+3 \text{ divides } 2(5n+26)-5(2n+3)=37$$

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