◇◇ **Graduate Course Description** ◇◇

Fall 1999

Additive and Multiplicative Problems in Number Theory, M788K

Cool problems & slick techniques

Instructor: K. Ford

Prerequisites: A graduate level course in number theory, or consent of the instructor.

Course Description. Additive number theory is concerned with the following type of problem: Let A and B be sets of non-negative integers, and let $A+B$ be the set of integers which can be written as a sum of an element of A and an element of B. Given A and B, what are the characteristics of $A+B$? The sets A and B can be general sets or special sets such as the set of primes, the set of squares, the set of square-free numbers, etc. One common question is to ask whether or not a given set A is a "basis", i.e. does there exist a number s so that every positive integer (or every integer beyond some point) can be written as a sum of s members of A.

Many of the most famous problem in number theory (and in all of mathematics) are of this type. Examples include the Twin Prime Conjecture, Goldbach's Conjecture and Waring's problem. Below are some of the theorems that will be proved:

(0) All sets with "positive density" are a basis.
(1) The set P of primes is a basis.
(2) All large odd numbers are a sum of three primes.
(3) All large integers are the sum of seven positive cubes.
(4) A "positive fraction" of numbers are the sum of a prime and a power of 2.

Multiplicative number theory includes various problems about the prime factors of numbers and the distribution of arithmetic functions such as the number of divisors function, the sum of divisors function, and Euler's ϕ-function. We will be concerned with the following type of problem: given an arithmetic function $f(n)$, what is the average value of the $f(n)$ over the integers from 1 to N, what is the "normal order" of the function and the extremal values of the function over the same set of numbers? The average and "normal" orders may be different, as seen with the number of divisors function $d(n)$. We will prove that over the numbers n from 1 to N, $d(n)$ has average value about $\log N$ but "normal" order about $(\log N)^{\log 2}$.

The methods we shall develop to study such problems will range from elementary (Schnirelmann theory, combinatorial sieves) to analytic (circle method). Some analytic tools, such as the Prime Number Theorem, will be used but not proved.