# MATH 782: ANALYTIC NUMBER THEORY

## COURSE DESCRIPTION:

Analytic Number Theory concerns itself with the use of analysis to obtain information about the natural numbers. For example, we might ask, "How many primes are there?" A completely elementary argument can be given that there exist infinitely many primes. A more analytic proof establishes the stronger result that the sum of the reciprocals of the primes $\sum_{p} 1/p$ diverges. One could then seek to establish even more information. In this regard (and this can be made precise) $\sum_{p \le x} 1/p$ is about the size of $\log\log x$. Returning to the original question, a more complete answer answer can be given. The number of primes up to $x$, denoted by $\pi(x)$, is "about" $x/\log x$. More precisely,

$$\lim_{x \to \infty} \frac{\pi(x)}{x/\log x} = 1.$$

This is known as the Prime Number Theorem. Some results that will be obtained in this course include:

**(1)** Results (as above) related to the distribution of primes will be a main focus of this course. In particular, a proof of the Prime Number Theorem will be given.

**(2)** Another classical result on the subject that will be established is Dirichet's Theorem. It asserts that if $a$ and $b$ are relatively prime positive integers, then there are infinitely many primes of the form $an + b$ (with $n$ an integer).

**(3)** A twin prime is a prime which differs from another prime by 2. It is unknown whether or not there exist infinitely many twin primes. We will establish Brun's theorem that the sum of the recipricals of the twin primes converges.

**(4)** Let $R$ be a rectangle, and suppose $R$ can be expressed as a union of rectangles $R_j$ with edges parallel to $R$ and common points only along these edges. Suppose each $R_j$ has at least one edge of integer length. We'll show $R$ itself must have an edge of integer length.

**(5)** Suppose we have a finite system of congruences

$$x \equiv a_j \pmod{m_j} \quad (j = 1, 2, \dots, r)$$

having the property that each integer satisfies exactly one of these congruences. Let $m = \max\{m_j : 1 \le j \le r\}$. We will establish that there are at least two values of $j \in \{1, 2, \dots, r\}$ such that $m_j = m$.