MATH 780: ELEMENTARY NUMBER THEORY

COURSE DESCRIPTION:

This is an introductory course in Elementary Number Theory. The first two thirds of the class will include such topics as rational and irrational numbers, the division algorithm, the Euclidean Algorithm, continued fractions, modulo arithmetic, Fermat's Little Theorem, public-key encryptions, the Chinese Remainder Theorem, primitive roots, and quadratic reciprocity. These topics will be introduced and various applications will be given to show how they relate to the subject of Number Theory as a whole. As examples of applications to be discussed, we mention the following questions:

(1) When the number 123456789101112...1979 is divided by 1980, what is the remainder?

(2) Is there is an odd integer > 1 which is not the sum of a prime and a power of 2?

(3) Imagine sitting at the origin in the plane and looking out at all the lattice points (i.e., those points (x, y) where both x and y are integers). Imagine too that you have very good eyes that cannot only see these objects with no dimensions but also can see points that are arbitrarily far away, even $(1, 10^{1000})$, which even you must agree is quite impressive. But alas you do not have x-ray vision. Some lattice points are hidden by others, and so you cannot see them. For example, $(1, 1)$ is blocking your view of $(2, 2)$ and $(2, 3)$ is making it impossible for you to see $(4, 6)$. Are there arbitrarily large areas in the plane, say enclosed by arbitrarily big circles, all of whose lattice points are hidden from your view?

(4) Are there any squares which are eleven more than a cube?

The last third of the course will be devoted to discussions of asymptotics, Riemann-Stieltjes integrals, elementary results related to the distribution of primes, and sieve techniques. In particular, we will show that the sum of the reciprocals of the primes diverges but the sum of the reciprocals of the twin primes (primes like 29 and 31 which differ by 2) converges. We will also discuss how one can show that Fermat's Last Theorem holds for almost all exponents. Throughout the course, some emphasis will be placed on open problems which show the extent of what researchers can and cannot do thus far in Number Theory.