MATH 241: TEST 3

Name ______________________________

Instructions and Point Values: Put your name in the space provided above. Check that your test contains 6 different pages including one blank page. Work each problem below and show ALL of your work. Do NOT use a calculator.

Problem (1) is worth 20 points.
Problem (2) is worth 16 points.
Problem (3) is worth 16 points.
Problem (4) is worth 16 points.
Problem (5) is worth 16 points.
Problem (6) is worth 16 points.

---

(1) Calculate the following double integrals. **SIMPLIFY** your answers.

(a) $\displaystyle\int_1^2 \int_0^y xy\,dx\,dy$

(1) (continued)

(b) $\int_0^{\pi/2} \int_0^1 r \cos\theta \, dr \, d\theta$

(c) $\int_0^{\pi} \int_{x^2}^{\pi^2} \frac{\sin\sqrt{y}}{y} \, dy \, dx$ (Hint: You cannot evaluate $\int \frac{\sin\sqrt{y}}{y} \, dy$.)

(2) Let $\overrightarrow{F} = z^2\,\mathbf{i} + y^2\,\mathbf{j} + x^2\,\mathbf{k}$. Calculate the divergence and curl of $\overrightarrow{F}$.

Divergence: $\boxed{}$

Curl: $\boxed{}$

(3) Calculate cylindrical coordinates (r, θ, z) and spherical coordinates (ρ, ϕ, θ) for the point with rectangular coordinates $(x, y, z) = (1, 1, \sqrt{6})$. **<u>SIMPLIFY</u>** your answers (your answers should NOT involve inverse trignometric functions).

(r, θ, z): $\boxed{}$

(ρ, ϕ, θ): $\boxed{}$

(4) Rewrite the following integral using polar coordinates and then calculate its value.

$$\int_{-4}^{4} \int_{0}^{\sqrt{16-y^2}} e^{(x^2+y^2)}\, dx\, dy$$

(5) Write a triple integral in spherical coordinates which represents the volume of the solid inside the sphere $x^2 + y^2 + z^2 = 9$ and *above* the surface $z = -\sqrt{x^2 + y^2}$. (Do not evaluate the integral.)

(6) Calculate

$$\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_{4}^{\sqrt{25-x^2-y^2}} \frac{\sin\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}\,(\sqrt{25-x^2-y^2}-4)}\;dz\,dy\,dx.$$