Math 241: Test 2

Name ______________________________

Instructions and Point Values: Put your name in the space provided above. Make sure that your test has six different pages including one blank page. Work each problem below and show ALL of your work. You do not need to simplify your answers. Do NOT use a calculator.

Point Values: Problem (1) is worth 14 points, Problem (2) is worth 12 points, Problem (3) is worth 12 points, Problem (4) is worth 12 points, Problem (5) is worth 14 points, Problem (6) is worth 18 points, and Problem (7) is worth 18 points.

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(1) For both parts of this problem, $f(x, y) = x^2y^2 + x + y$.

(a) Find the directional derivative of $f(x, y)$ at the point $(2, 1)$ in the direction of $\langle 0, 3\rangle$.

Directional Derivative: $\boxed{}$

(b) There are infinitely many different values for the directional derivative of $f(x, y)$ at the point $(2, 1)$ (since there are infinitely many directions that can be used to compute the directional derivative). Which of these is maximal? In other words, what is the largest value of the directional derivative of $f(x, y)$ at the point $(2, 1)$?

Maximal Directional Derivative: $\boxed{}$

(2) Calculate $\dfrac{\partial^5 f(x,y)}{\partial^2 x\, \partial^3 y}$ where $f(x,y) = xy^2 \sin(x^2) + 4y^3 + \sqrt{x}$.

Answer: $\boxed{}$

(3) Using the Chain Rule, compute $\dfrac{\partial w}{\partial t}$ where

$$w = x^2 + xyz + x + 2z, \qquad x = t\sin(\sqrt{s}) + 2^s - t^2$$

$$y = 2s + s^2\sin(t), \qquad \text{and} \qquad z = t^2 s^3 - 2t$$

You do not need to put your answer in terms of s and t (the variables x, y, and z can appear in your answer).

Answer: $\boxed{}$

(4) Find an equation for the tangent plane to the surface $z = x^2y + xy^2 - 4$ at the point $(1, 2, 2)$.

Equation of Tangent Plane: $\boxed{}$

(5) Using the method of Lagrange multipliers, determine the maximum and minimum values of $f(x, y) = xy$ given the constraint $4x^2 + y^2 = 8$.

Maximum Value: $\boxed{}$

Minimum Value: $\boxed{}$

(6) Find the critical points of the function $f(x, y) = 3x + xy^2$ where (x, y) is restricted to points in the set $S = \{(x, y) : x^2 + y^2 \leq 9\}$. Also, determine the maximum and minimum values of $f(x, y)$ in S as well as all points (x, y) where these extreme values occur.

Critical Points: $\boxed{}$

The Maximum Value is $\boxed{}$ and it occurs at the pont(s) $\boxed{}$.

The Minimum Value is $\boxed{}$ and it occurs at the pont(s) $\boxed{}$.

(7) Let

$$f(x, y) = x^4 + 4xy + xy^2.$$

The function $f(x, y)$ has 3 critical points. Calculate the three critical points and indicate (with justification) whether each determines a local maximum value of $f(x, y)$, a local minimum value of $f(x, y)$, or a saddle point of $f(x, y)$.

1) First Critical Point: $\boxed{}$

Indicate Local Max, Local Min, or Saddle Point: $\boxed{}$

2) Second Critical Point: $\boxed{}$

Indicate Local Max, Local Min, or Saddle Point: $\boxed{}$

3) Third Critical Point: $\boxed{}$

Indicate Local Max, Local Min, or Saddle Point: $\boxed{}$