MATH 241: TEST 2

Name ______________________________

Instructions and Point Values: Put your name in the space provided above. Make sure that your test has six different pages including one blank page. Work each problem below and show ALL of your work. You do not need to simplify your answers. Do NOT use a calculator.

Point Values: Problem (1) is worth 12 points, Problem (2) is worth 12 points, Problem (3) is worth 14 points, Problem (4) is worth 12 points, Problem (5) is worth 14 points, Problem (6) is worth 18 points, and Problem (7) is worth 18 points.

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(1) For both parts of this problem, $f(x, y) = x^2 - y^2 + 1$.

(a) Find the directional derivative of $f(x, y)$ at the point $(0, 1)$ in the direction of $\langle 1, 1\rangle$.

DIRECTIONAL DERIVATIVE: $\boxed{}$

(b) There are infinitely many different values for the directional derivative of $f(x, y)$ at the point $(0, 1)$ (since there are infinitely many directions that can be used to compute the directional derivative). Which of these is maximal? In other words, what is the largest value of the directional derivative of $f(x, y)$ at the point $(0, 1)$?

MAXIMAL DIRECTIONAL DERIVATIVE: $\boxed{}$

(2) Calculate $\int_0^2 \int_1^2 (2xy^2 + y)\, dx\, dy.$

ANSWER: $\boxed{}$

(3) Find an equation for the tangent plane to the surface $x^3 - x\sin(y) + z^2 = 0$ at the point $(-1, 0, 1)$.

EQUATION OF TANGENT PLANE: $\boxed{}$

(4) Using the Chain Rule, compute $\dfrac{\partial z}{\partial t}$ where

$$z = xy^2 + x^2 + 3y + 5, \qquad x = s^2 t + (s+3)^2 e^{2s+1},$$

and

$$y = s(s-1)^4 + \cos(4s+1) + t^2.$$

You do not need to put your answer in terms of s and t (the variables x and y can appear in your answer).

ANSWER: $\boxed{}$

(5) Using the method of Lagrange multipliers, determine the minimum value of $f(x,y) = x^2 + y^2 - x + y - 1$ given the constraint $x - y + 1 = 0$.

MINIMUM VALUE: $\boxed{}$

(6) For both parts of this problem, consider

$$f(x, y) = 9x^2 + 6y^2 + 6x + 4.$$

(a) Determine the global maximum and the global minimum value of $f(x, y)$ on the circle $x^2 + y^2 = 4$. (This problem works well with or without Lagrange multipliers.)

GLOBAL MAXIMUM VALUE: $\boxed{}$

GLOBAL MINIMUM VALUE: $\boxed{}$

(b) Let $R = \{(x, y) : x^2 + y^2 \leq 4\}$, so R is the circle centered at the origin of radius 2 together with its interior. Determine the global maximum and the global minimum value of $f(x, y)$ on R.

GLOBAL MAXIMUM VALUE: $\boxed{}$

GLOBAL MINIMUM VALUE: $\boxed{}$

(7) Let

$$f(x,y) = 2x^2y - 8xy + y^2 + 5.$$

The function $f(x,y)$ has 3 critical points. Calculate the critical points and indicate (with justification) whether each determines a local maximum value of $f(x,y)$, a local minimum value of $f(x,y)$, or a saddle point of $f(x,y)$.

1) FIRST CRITICAL POINT: $\boxed{}$

LOCAL MAX, LOCAL MIN, OR SADDLE PT: $\boxed{}$

2) SECOND CRITICAL POINT: $\boxed{}$

LOCAL MAX, LOCAL MIN, OR SADDLE PT: $\boxed{}$

3) THIRD CRITICAL POINT: $\boxed{}$

LOCAL MAX, LOCAL MIN, OR SADDLE PT: $\boxed{}$