MATH 241: TEST 2

Name ______________________________

Instructions and Point Values: Put your name in the space provided above. Work each problem below and show ALL of your work. You do not need to simplify your answers. Do NOT use a calculator.

Problem (1) is worth 12 points.
Problem (2) is worth 14 points.
Problem (3) is worth 14 points.
Problem (4) is worth 14 points.
Problem (5) is worth 14 points.
Problem (6) is worth 16 points.
Problem (7) is worth 16 points.

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(1) Let

$$f(x,y) = \frac{x^3 y}{x^4 + y^4}.$$

Does $\lim_{(x,y)\to(0,0)} f(x,y)$ exist? If so, what is it? If not, why not?

(2) Let $\vec{v} = \langle 1, 3 \rangle$ and $f(x,y) = x^2 y + y^2$. Calculate the directional derivative of $f(x,y)$ in the direction $\vec{v}$ at the point $(1, -1)$.

(3) Find an equation for the tangent plane to the surface $x^2 - 2y^2 = xyz^2$ at the point $(1, -1, -1)$.

(4) Let $R = \{(x, y) : -1 \leq x \leq 3,\ 0 \leq y \leq 2\}$ and

$$f(x, y) = \begin{cases} 3 & \text{for } -1 \leq x \leq 2,\ 0 \leq y \leq 2 \\ 4 & \text{for } 2 < x \leq 3,\ 0 \leq y \leq 2. \end{cases}$$

Evaluate $\iint\limits_R f(x, y)\, d\, A$.

(5) Let $z = x^3y + y^2x - x$ where $x = e^s t$ and $y = te^{st}$. Using the Chain Rule, calculate $\partial z/\partial s$ in terms of s and t (but you do not need to simplify your answer).

(6) Let $R = \{(x, y) : x^2 + y^2 \leq 4\}$, so R is the circle centered at the origin of radius 2 together with its interior. Let $f(x, y)$ be defined on R by

$$f(x, y) = 6x^2 + 3y^2 - 6x - 9.$$

Calculate the global maximum value and the global minimum value of $f(x, y)$ on R.

GLOBAL MAXIMUM VALUE: $\boxed{}$

GLOBAL MINIMUM VALUE: $\boxed{}$

(7) Let

$$f(x, y) = (3y^4 + 1)(x^2 - 2x + 2) - 12y^3 + 12y^2.$$

The function $f(x, y)$ has 3 critical points. Calculate the critical points and indicate (with justification) whether each determines a local maximum value of $f(x, y)$, a local minimum value of $f(x, y)$, or a saddle point of $f(x, y)$.

1) FIRST CRITICAL POINT: $\boxed{}$

LOCAL MAX, LOCAL MIN, OR SADDLE PT: $\boxed{}$

2) SECOND CRITICAL POINT: $\boxed{}$

LOCAL MAX, LOCAL MIN, OR SADDLE PT: $\boxed{}$

3) THIRD CRITICAL POINT: $\boxed{}$

LOCAL MAX, LOCAL MIN, OR SADDLE PT: $\boxed{}$