Math 241: Calculus III Name ______________________

Test #2

Show All Work

Points: (1) 6 pts each part, (2) – (5) 13 pts each, (6) 18 pts

(1) (a) Calculate $\dfrac{\partial f}{\partial y}$ where $f(x,y) = x^2 \sin(xy)$.

(b) Calculate $\nabla f(2,1)$ where $f(x,y) = x^2 + y^3$.

(c) Calculate f_{xxxxyy} if $f(x,y) = x^3 y^2 \sqrt{y}\, \sin(y) e^y \ln y$.

(d) Integrate $\displaystyle\int_0^1 \int_0^3 xy^2 \, dx \, dy$

(e) Find the directional derivative of $f(x,y) = x^2 y + x + 2$ at the point $P = (1,1)$ in the direction of $\mathbf{v} = -i + j$.

(2) Find the equation of the tangent plane to the surface $z^2 = x^3 + y^2$ at the point $(2, 1, -3)$.

(3) Calculate $\frac{\partial z}{\partial t}$ given that $z = y^2\sqrt{y}\sin(x+y)$, $x = 3u + 2v$, $y = v^3 - 12v^2 + 5v + 16$, $u = r + 2s$, $v = r^2 + 3r + 5$, $r = w^3 + \cos(w)$, and $s = 3tw$. Use any method you want. You do not need to write your answer in terms of w and t (i.e., you may have other variables in your answer). Do NOT simplify.

(4) Find every point $P = (a, b, c)$ on the surface $z = (x + y)^3 x + x^2 - x$ such that the tangent plane to the surface at P is horizontal (i.e., the tangent plane is parallel to the xy–plane).

(5) Using the second derivative test for functions of two variables, find all points (a, b, c) where the graph of $f(x, y) = x^2 + 2xy + 2y^2 + 2x + 1$ has a local maximum or a local minimum. For each such point, indicate which (a local maximum or a local minimum) occurs.

(6) Find the maximum and minimum values for the function $f(x, y) = xy^2 + 3y^2 + 5x - 5$ in the disk $x^2 + y^2 \leq 4$. Be sure you justify your answers.

Maximum: [ ]

Minimum: [ ]