MATH 241: TEST 1, Spring 2001

Name ______________________________

Instructions: Put your name in the space provided above. Check that your copy of this test contains 8 different pages (including a page with graphs and a blank page). Work each problem and show ALL of your work. Unless stated otherwise, you do not need to simplify your answers. Do NOT use a calculator.

Point Values: Problems (1), (2), (3), and (4) are each worth 10 points. Problems (5), (6), and (7) are each worth 14 points. Problem (8) is worth 18 points. Parts of problems will be weighted equally.

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(1) If $P = (-1, 4, 2)$ and $Q = (2, -2, 6)$, then what is the vector $\overrightarrow{PQ}$?

$\overrightarrow{PQ} =$ [ ] (express using its components)

(2) What is the area of the triangle having vertices $(0, 0, 0)$, $(1, 0, 1)$, and $(1, -2, 0)$?

Area: [ ]

(3) Let $P = (0, 0, 0)$, $Q = (1, 1, 0)$, $R = (-2, 0, 3)$, and $S = (1, 2, 1)$. What is the volume of a parallelepiped having three edges given by the line segments $\overline{PQ}$, $\overline{PR}$, and $\overline{PS}$?

Volume: [ ]

(4) Let $\vec{r}(t) = \langle t^2, -t^3, 2t^2 + 2t - 1 \rangle$. Then at $t = 1$, $\vec{r} = \langle 1, -1, 3 \rangle$. Find a unit vector that is tangent to the curve given by $\vec{r}(t)$ at $t = 1$ (that is if its tail is placed on the curve at the point $(1, -1, 3)$).

Unit Vector: [ ]

(5) Let a particle's position vector at time t be given by $\vec{r}(t) = \langle \cos^2 t, \cos t \sin t, 2\sqrt{2}\,t \rangle$.

(a) Show that its speed is a constant.

(b) Calculate the length of the curve traced by $\vec{r}(t)$ from $t = 0$ to $t = 1$. (You should at least explain what you would do if you could not do part (a).)

Length of Curve: [ ]

(6) (a) Let $\mathcal{P}$ be the plane $x - y + 2z = 3$. Find the parametric equations of the line ℓ that is perpendicular to $\mathcal{P}$ and passes through the point $(0, 0, 1)$.

Parametric Equations:

(b) Find the point of intersection of the plane $\mathcal{P}$ and the line ℓ given in part (a).

Point of Intersection:

(7) (a) Find the point where the lines below intersect. Justify your answer.

$$\ell_1 : \begin{cases} x = 1 + t \\ y = 2 - t \\ z = t \end{cases} \qquad \ell_2 : \begin{cases} x = t \\ y = 5 + t \\ z = -1 + t \end{cases}$$

Point of Intersection: [ ]

(b) Calculate the angle (that is, the smallest angle) between the two lines in part (a).

Angle: [ ] (*use* an inverse trigonometric function)

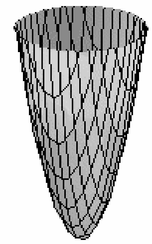
(8) The graphs for the equations below are similar to the graphs on the next page. The orientation and the scaling may be different. For each equation, indicate which graph on the next page best matches it. For example, if the equation is for a hyperbolic paraboloid, then the graph you choose should be a hyperbolic paraboloid. Indicate your choice by putting the corresponding letter from the next page after the equation below. Next, read the question on the next page corresponding to the graph you choose. Then go back to the equation below and answer the question for the graph of that equation. Do **NOT** answer the question for the graph on the next page (since it may be oriented differently than the graph of the equation below).

(i) $4x^2 + y^2 + z^2 = 16$

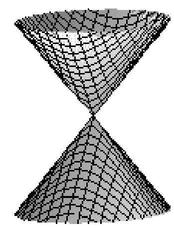
(ii) $4x^2 - y^2 + z^2 = 16$

(iii) $4x^2 - y^2 + z = 16$

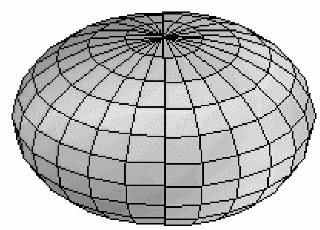
(a) This is a graph of an elliptic paraboloid. Where is the vertex of the paraboloid in the graph on the previous page? Answer with a point (give all 3 coordinates).



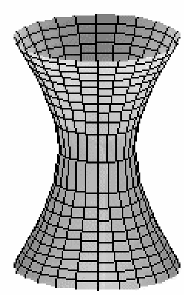
(b) This is an elliptic cone. What is the intersection of the graph for the corresponding equation on the previous page with the plane $y = 2001$? Answer either lines, an ellipse, a square, a parabola, a hyperbola, or a chessboard.



(c) This is an ellipsoid. Indicate where the ellipsoid intersects the y-axis for the corresponding graph on the previous page. Your answer should be two points. (Give all 3 coordinates for each point.)



(d) This is a graph of a hyperboloid of 1 sheet. There are two points on the corresponding graph on the previous page that are closer (a shorter distance) to the y-axis than the other points on the graph. Tell me one of these points (give all 3 coordinates).



(e) This is a graph of a hyperbolic paraboloid. There is a plane, perpendicular to one of the x, y, or z-axes, that intersects the graph of the hyperbolic paraboloid from the previous page in two lines. What is the equation of the plane?