MATH 241: TEST 1, Fall 2001

Name ______________________________

Instructions: Put your name in the space provided above. Check that your copy of this test contains 9 different pages (including a page with graphs and a blank page). Work each problem and show ALL of your work. Unless stated otherwise, you do not need to simplify your answers. Do NOT use a calculator.

Point Values: Problems (1) & (6) are worth 12 points each, Problem (2) is worth 10 points, Problem (3) is worth 14 points, Problem (4) is worth 15 points, and Problems (5) & (7) are worth 18 points each. Your name is worth one point.

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(1) (a) If $P = (1, 0, -1)$ and $Q = (3, -2, 1)$, then what is the vector $\overrightarrow{PQ}$?

$\overrightarrow{PQ} = \boxed{}$ (express using its components)

(b) Calculate $\langle 3, -4, 1\rangle - 2\langle 1, 2, -1\rangle$.

Answer: $\boxed{}$

(c) If $R = (\sqrt{34}, \pi - 17, 789)$, then what is the value of $\overrightarrow{PQ} + \overrightarrow{QR} + \overrightarrow{RP}$? (Hint: You may want to give a little thought to the question, before doing any arithmetic.)

$\overrightarrow{PQ} + \overrightarrow{QR} + \overrightarrow{RP} = \boxed{}$ (the answer is a vector; write its components)

(2) (a) What is the area of the triangle with vertices $P = (-1, 0, 2)$, $Q = (0, 0, 3)$, and $R = (7, 4, 3)$? Simplify your answer.

Area: $\boxed{}$

(b) If $S = (0, 0, 1)$, then what is the volume of the parallelepiped having as edges the three segments $\overline{PQ}$, $\overline{PR}$, and $\overline{PS}$?

Volume: $\boxed{}$

(3) (a) Let $\vec{u} = \langle x_1, x_2, x_3 \rangle$, $\vec{v} = \langle y_1, y_2, y_3 \rangle$, and $\vec{w} = \langle z_1, z_2, z_3 \rangle$. What's the value of $\vec{w} \cdot (\vec{u} + \vec{v})$ in terms of the components of the vectors?

Answer: $\boxed{}$

(b) Let $\vec{u} = \langle x_1, x_2, x_3 \rangle$, $\vec{v} = \langle y_1, y_2, y_3 \rangle$, and $\vec{w} = \langle z_1, z_2, z_3 \rangle$. What's the value of $\vec{w} \cdot \vec{u} + \vec{w} \cdot \vec{v}$ in terms of the components of the vectors?

Answer: $\boxed{}$

(c) Explain briefly what (a) and (b) have to do with the equation

$$(\vec{u} + \vec{v}) \cdot (\vec{u} + \vec{v}) = \vec{u} \cdot \vec{u} + 2\vec{u} \cdot \vec{v} + \vec{v} \cdot \vec{v}.$$

(d) If $|\vec{u}| = 4$, $|\vec{v}| = 3$, and $|\vec{u} + \vec{v}| = 6$, then what's the value of $\cos\theta$ where θ is the angle between $\vec{u}$ and $\vec{v}$? (Hint: Use part (c).)

Answer: $\boxed{}$ (answer a specific number)

(4) (a) Let $\vec{r}(t) = \langle t^2 \sin(t), -t^2 \cos(t), 2t \rangle$ be the position vector of a moving particle at time t. Calculate the velocity $\vec{v}(t)$ for the particle.

$\vec{v}(t) =$ $\boxed{}$

(b) Calculate the speed of the particle in part (a). Your answer should be in terms of t. Simplify your answer.

Speed: $\boxed{}$

(c) Calculate the length of the curve traced by the moving particle from time $t = 0$ to $t = 3$.

Length of Curve: $\boxed{}$

(5) (a) The points that are equidistant from $(-1, 3, 2)$ and $(3, 1, 2)$ form a plane. Calculate an equation for this plane.

Equation of Plane: $\boxed{}$

(b) Find the point where the lines below intersect. Justify your answer.

$$\ell_1 : \begin{cases} x &= 3t \\ y &= t \\ z &= -1+t \end{cases} \qquad \ell_2 : \begin{cases} x &= t \\ y &= 2-t \\ z &= -2+t \end{cases}$$

Point of Intersection: $\boxed{}$

(6) Find an equation of a plane through point P and parallel to line ℓ given below.

$$P = (2, -1, 1) \qquad \text{and} \qquad \ell : \begin{cases} x &= \quad t \\ y &= \quad -t \\ z &= 2 - t \end{cases}$$

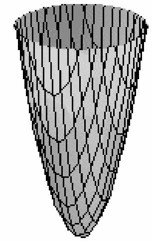
(7) The graphs for the equations below are similar to the graphs on the next page. The orientation and the scaling may be different. For each equation, indicate which graph on the next page best matches it. For example, if the equation is for a hyperbolic paraboloid, then the graph you choose should be a hyperbolic paraboloid. Indicate your choice by putting the corresponding letter from the next page after the equation below. Next, read the question on the next page corresponding to the graph you choose. Then go back to the equation below and answer the question for the graph of that equation. Do **NOT** answer the question for the graph on the next page (since it may be oriented differently than the graph of the equation below).

(i) $x^2 - y^2 + z^2 + 1 = 0$

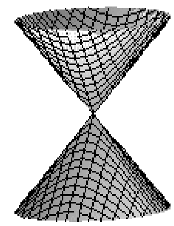
(ii) $x^2 - y^2 + z + 1 = 0$

(iii) $x^2 - y + z^2 + 1 = 0$

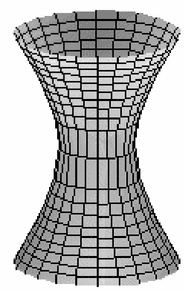
(a) This is a graph of an elliptic paraboloid. Where is the vertex of the paraboloid in the graph on the previous page? Answer with a point (give all 3 coordinates).



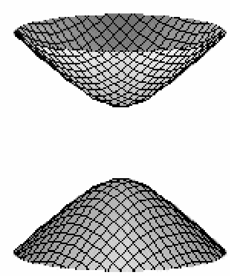
(b) This is an elliptic cone. What is the intersection of the graph for the corresponding equation on the previous page with the plane $y = 2001$? Answer either lines, an ellipse, a square, a parabola, a hyperbola, or a chessboard.



(c) This is a graph of a hyperboloid of 1 sheet. There are two points on the corresponding graph on the previous page that are closer (a shorter distance) to the y-axis than the other points on the graph. Tell me one of these points (give all 3 coordinates).



(d) This is a graph of a hyperboloid of 2 sheets. The surface for the corresponding graph on the previous page intersects one of the x-axis, y-axis, and z-axis. Which axis does it intersect?



(e) This is a graph of a hyperbolic paraboloid. There is a plane, perpendicular to one of the x, y, or z-axes, that intersects the graph of the hyperbolic paraboloid from the previous page in two lines. What is the equation of the plane?