MATH 241: TEST 1

Name _____

Instructions: Put your name in the space provided above. Check that your copy of this test contains 6 different pages. Work each problem and show ALL of your work. Unless stated otherwise, you do not need to simplify your answers. Do NOT use a calculator.

Point Values: Problem (1) is worth 28 points (7 points each part), Problem (2) is worth 20 points (10 points each part), Problem (3) is worth 12 points, Problem (4) is worth 22 points (11 points each part), and Problem (5) is worth 18 points (6 points each part). There is an Extra Credit Problem on the last page.

---

(1) Throughout this problem, $P = (1, 3, -1)$ and $Q = (-1, 0, 5)$.

(a) Calculate the distance from P to Q.

Distance: [ ]

(b) Determine the midpoint of the line segment with endpoints P and Q.

Midpoint: [ ] (use correct notation)

(c) What is the vector going from P to Q? Express your answer in the form $\langle a, b, c \rangle$.

Vector: [ ]

(d) What is the length (or magnitude) of the vector $\overrightarrow{PQ}$ (the vector in part (c))?

Length: [ ]

(2) Throughout this problem, $P = (1, 4, 8)$, $Q = (0, 0, 0)$, and $R = (0, -6, -8)$.

(a) Calculate the area of ΔPQR (the triangle with vertices P, Q, and R).

Area: $\boxed{}$

(b) Calculate the measure of the angle $\angle PQR$. (Your answer **SHOULD** make use of an inverse trigonometric function.)

Measure of Angle: $\boxed{}$

(3) Calculate the third order partial derivative indicated below.

$$\frac{\partial^3}{\partial x\, \partial y^2} \left(y + xy(\cos x)(\sin x) - \sin(xy)\right)$$

Answer: $\boxed{}$

(4) (a) Determine the parametric equations for a line that is perpendicular to the plane $2x + y - z = 4$ and that passes through the point $(-1, 2, 0)$.

(b) Determine the parametric equations for a line that is *parallel* to the plane $2x+y-z = 4$ and that passes through the point $(-1, 2, 0)$. (There is more than one correct answer.)

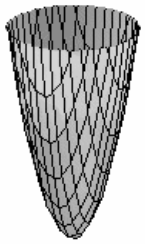
(5) The graphs for the equations below are similar to the graphs on the next page. The orientation and the scaling may be different. For each equation, indicate which graph on the next page best matches it. For example, if the equation is for a hyperbolic paraboloid, then the graph you choose should be a hyperbolic paraboloid. Indicate your choice by putting the corresponding letter from the next page after the equation below. Next, read the question on the next page corresponding to the graph you choose. Then go back to the equation below and answer the question for the graph of that equation. Do **NOT** answer the question for the graph on the next page (since it may be oriented differently than the graph of the equation below).

(i) $x^2 + 2y^2 + z^2 - 9 = 0$

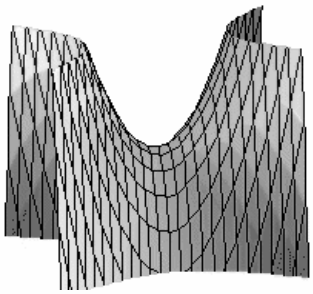
(ii) $x^2 - 2y^2 + 3z^2 + 4 = 0$

(iii) $x^2 - y - z^2 = 0$

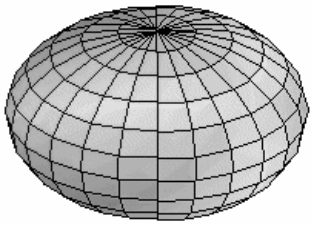
(a) This is a graph of an elliptic paraboloid. Where is the vertex of the paraboloid in the graph on the previous page? Answer with a point (all 3 coordinates).



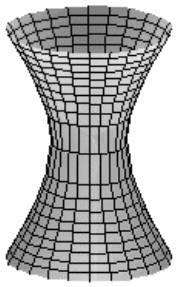
(b) This is a hyperbolic paraboloid. What is the intersection of the graph for the corresponding equation on the previous page with the plane $y = -10$? Answer either a line, an ellipse, a square, a parabola, or a hyperbola.



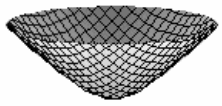
(c) This is an ellipsoid. If a plane parallel to the xy-plane intersects the graph for the corresponding equation on the previous page at more than one point, then what will that intersection be? Answer either a line, an ellipse, a square, a parabola, or a hyperbola.

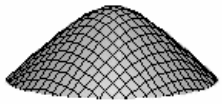


(d) This is a graph of a hyperboloid of 1 sheet. What is the intersection of the corresponding graph on the previous page with planes parallel to the yz-plane? Answer either lines, ellipses, squares, parabolas, or hyperbolas.



(e) This is a graph of a hyperboloid of 2 sheets. What are the vertices for the corresponding graph on the previous page? (Give the three coordinates for each vertex.)





EXTRA CREDIT: (For a possible 5 additional points.)

The plane $x + z = 0$ intersects the surface in Problem (5)(iii) in a line. Determine the parametric equations for this line (with justificiation).