MATH 241: TEST 1

Name ______________________________

Instructions: Put your name in the space provided above. Check that your copy of this test contains 6 different pages. Work each problem and show ALL of your work. Unless stated otherwise, you do not need to simplify your answers. Do NOT use a calculator.

Point Values: Each of Problems (1) through (9) is worth 8 points. Problem (10) is worth 13 points, and Problem (11) is worth 15 points.

---

(1) Calculate the distance from the point $P = (0, 1, 2)$ to the point $Q = (3, 1, -2)$.

Answer: $\boxed{}$

(2) Calculate the midpoint of the line segment $\overline{XY}$ where $X = (4, 3, 5)$ and $Y = (2, 5, -3)$.

Answer: $\boxed{}$

(3) Calculate the dot product of the vectors $\overrightarrow{\mathbf{u}} = \langle -2, 3, 2 \rangle$ and $\overrightarrow{\mathbf{v}} = \langle 4, -1, 1 \rangle$.

Answer: $\boxed{}$

(4) Calculate the angle θ in $[0, \pi]$ between the vectors $\overrightarrow{\mathbf{u}} = \langle 1, -2, 2 \rangle$ and $\overrightarrow{\mathbf{v}} = \langle 1, 0, 1 \rangle$. Simplify your answer so that it does not involve inverse trignometric functions.

Answer: $\boxed{}$

(5) Calculate the projection $\mathrm{proj}_{\overrightarrow{\mathbf{v}}} \overrightarrow{\mathbf{u}}$ where $\overrightarrow{\mathbf{u}} = \langle 1, -2, 2 \rangle$ and $\overrightarrow{\mathbf{v}} = \langle 1, 0, 1 \rangle$. Your answer should be a vector.

Answer: $\boxed{}$

(6) One level curve for $z = \left(x^2 + y^2\right)^{3/2}$ is a point (when $z = 0$). Describe the other level curves for $z = \left(x^2 + y^2\right)^{3/2}$. Your answer should be either lines, circles, parabolas, hyperbolas, or parallelepipeds. Be sure to justify your answer.

Answer: $\boxed{}$

(7) Calculate $\dfrac{\partial^2}{\partial x\, \partial y}\left(\dfrac{\sin(xy)}{x}\right)$.

Answer: $\boxed{}$

(8) A curve is described by the position vector $\overrightarrow{\mathbf{r}}(t) = t^2\,\mathbf{i} - 5\,\mathbf{j} + \sqrt{t}\,\mathbf{k}$. Calculate a unit tangent vector for this curve at $t = 1$.

Answer: $\boxed{}$

(9) Three vertices of a parallelogram are $P = (2, 1, 5)$, $Q = (-3, 2, 5)$, and $R = (-3, 1, 6)$. Calculate the area of the parallelogram.

Answer: $\boxed{}$

(10) The plane $\mathcal{P}_1$ has the equation $x + 2z = 4$, and the plane $\mathcal{P}_2$ has the equation $x - y = 2$. They intersect in a line ℓ. Calculate the equation of the **PLANE** containing the line ℓ and the point $(1, 1, 1)$.

Answer: $\boxed{}$

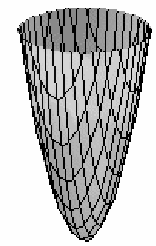
(11) The graphs for the equations below are similar to the graphs on the next page. The orientation and the scaling may be different. For each equation, indicate which graph on the next page best matches it. For example, if the equation is for a hyperbolic paraboloid, then the graph you choose should be a hyperbolic paraboloid. Indicate your choice by putting the corresponding letter from the next page after the equation below. Next, read the question on the next page corresponding to the graph you choose. Then go back to the equation below and answer the question for the graph of that equation. Do **NOT** answer the question for the graph on the next page (since it may be oriented differently than the graph of the equation below).

(i) $x^2 - y^2 + 3z = 0$

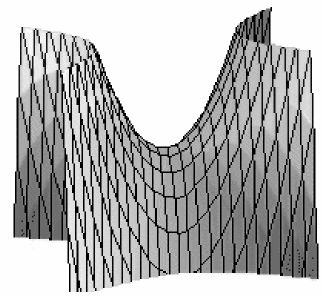
(ii) $x^2 + y^2 + 3z = 1$

(iii) $x^2 - y^2 + 3z^2 + 1 = 0$

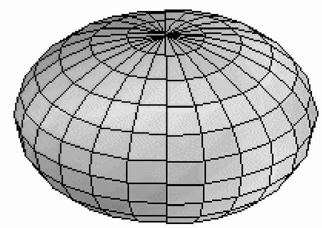
(a) This is a graph of an elliptic paraboloid. Where is the vertex of the paraboloid in the graph on the previous page? Answer with a point (all 3 coordinates).



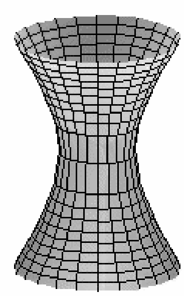
(b) This is a hyperbolic paraboloid. What is the intersection of the graph for the corresponding equation on the previous page with the $xy-$plane? Answer either lines, circles, squares, parabolas, or hyperbolas.



(c) This is an ellipsoid. At what two points does the ellipsoid intersect the z-axis in the graph for the corresponding equation on the previous page?



(d) This is a graph of a hyperboloid of 1 sheet. What is the intersection of the corresponding graph on the previous page with planes parallel to the $xy-$plane? Answer either lines, circles, squares, parabolas, or hyperbolas.



(e) This is a graph of a hyperboloid of 2 sheets. What are the vertices for the corresponding graph on the previous page? (Give the three coordinates for each vertex.)