Math 241: Final Exam

Name ______________________________

Instructions and Point Values: Put your name in the space provided above. Check that your test contains 11 different pages including one blank page. Work each problem below and show ALL of your work. Unless stated otherwise, you do not need to simplify your answers. Do NOT use a calculator.

There are 100 total points possible on this exam. The points for each problem in each part is indicated below.

PART I:

Problem (1) is worth 8 points.

Problem (2) is worth 6 points.

Problem (3) is worth 6 points.

Problem (4) is worth 7 points.

Problem (5) is worth 8 points.

Problem (6) is worth 9 points.

Problem (7) is worth 8 points.

PART II:

Problem (1) is worth 12 points.

Problem (2) is worth 12 points.

Problem (3) is worth 12 points.

Problem (4) is worth 12 points.

PART I.

(1) Let $\vec{u} = \langle 2, 1, -2 \rangle$ and $\vec{v} = \langle 4, -1, -1 \rangle$. Calculate

(a) $2\vec{u} - \vec{v}$

Answer: $\boxed{}$

(b) $|\vec{u}|$ (the magnitude of $\vec{u}$)

Answer: $\boxed{}$

(c) the angle between $\vec{u}$ and $\vec{v}$ (simplify so no inverse trig functions are in your answer)

Answer: $\boxed{}$

(d) $\vec{u} \times \vec{v}$

Answer: $\boxed{}$

(2) Calculate an equation for the tangent plane to the surface

$$xy + xz + yz = 3$$

at the point $(1, 1, 1)$.

Answer: $\boxed{}$

(3) For both parts of this problem, $f(x, y) = x^2y - y^3 + 2y$.

(a) Find the directional derivative of $f(x, y)$ at the point $(2, -1)$ in the direction of $\langle 1, 2\rangle$.

Directional Derivative: $\boxed{}$

(b) There are infinitely many different values for the directional derivative of $f(x, y)$ at the point $(2, -1)$. Find a unit vector $\vec{u}$ such that the directional derivative of $f(x, y)$ in the direction of $\vec{u}$ is maximal (as large as possible).

$\vec{u} =$ $\boxed{}$

(4) Let

$$f(x,y) = \lim_{h\to 0} \frac{(x+2y+h)^{3/2} - (x+2y)^{3/2}}{h}.$$

Calculate $f(2,1)$. (Comment: If you get the answer 6, then you probably have the basic idea right but not the answer.)

Answer: $\boxed{}$

(5) (a) Calculate the line integral

$$\int_{\mathcal{C}} (x^2 \cos y + y)\; dx \;+\; (x^2 \cos y - y)\; dy$$

where $\mathcal{C}$ is the line segment from $(1,0)$ to $(0,1)$.

Answer: $\boxed{}$

(b) What is the value of the line integral in part (a) if instead $\mathcal{C}$ is the line segment from $(0,1)$ to $(1,0)$?

Answer: $\boxed{}$

(6) Calculate each of the following integrals.

(a) $\displaystyle\int_0^2 \int_0^{3x} \int_y^{x+y} dz\, dy\, dx$

Answer:

(b) $\displaystyle\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} dz\, dx\, dy$

Answer:

(c) $\displaystyle\int_0^1 \int_x^{\sqrt{2-x^2}} (x^2+y^2)^{3/2}\; dy\, dx$

Answer:

(7) Let

$$f(x, y) = xy(x+2)\,\mathrm{e}^{-2y^2}.$$

Then

$$f_x = 2y(x+1)\,\mathrm{e}^{-2y^2}, \qquad f_y = -x(x+2)(2y-1)(2y+1)\,\mathrm{e}^{-2y^2}$$

$$f_{xx} = 2y\,\mathrm{e}^{-2y^2}, \qquad f_{yy} = 4xy(x+2)(4y^2-3)\,\mathrm{e}^{-2y^2},$$

and

$$f_{xy} = -2(x+1)(2y-1)(2y+1)\,\mathrm{e}^{-2y^2}.$$

The function $f(x, y)$ has four critical points. Calculate the four critical points and indicate (with justification) whether each determines a local maximum value of $f(x, y)$, a local minimum value of $f(x, y)$, or a saddle point of $f(x, y)$.

	Critical Point	Local Min., Local Max., or Saddle Pt.
First		
Second		
Third		
Fourth		

PART II.

(1) Let $\mathcal{P}$ be the plane given by the equation $x - y + z = 2$. The point $Q = (-1, 2, 1)$ is not on the plane $\mathcal{P}$. Find the equation of a plane which passes through the point Q and is *perpendicular* to the plane $\mathcal{P}$.

Answer: $\boxed{}$

(2) Using Green's Theorem, calculate the line integral

$$\int_{\mathcal{C}} \big(\cos x + \sin y - xy^3\big)\, dx + \big(x \cos y - x^2y^2 + \mathrm{e}^{y^2+1}\big)\, dy$$

where $\mathcal{C}$ is the rectangle oriented counter-clockwise with vertices $(0,0)$, $(1,0)$, $(1,3)$, and $(0,3)$. Simplify your answer.

Answer: $\boxed{}$

(3) Determine the maximum and the minimum values of the function

$$18x^2 - 6x + 3 - 24xy + 16y^2$$

on the triangle $\{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq x\}$. Simplify your answers.

Maximum Value: $\boxed{}$

Minimum Value: $\boxed{}$

(4) Calculate the volume of the solid lying above the xy-plane and inside the surfaces

$$z^2 = -1 + x^2 + y^2 \qquad \text{and} \qquad 3x^2 + 3y^2 + z^2 = 4.$$

Simplify your answer. (Hint: Express the volume as a sum of two integrals.)

Answer: $\boxed{}$