MATH 241: FINAL EXAM

Name ______________________________

Instructions and Point Values: Put your name in the space provided above. Check that your test contains 14 different pages including one blank page. Work each problem below and show ALL of your work. Unless stated otherwise, you do not need to simplify your answers. Do NOT use a calculator.

There are 300 total points possible on this exam. The points for each problem in each part is indicated below.

PART I

Problem (1) is worth 16 points.

Problem (2) is worth 12 points.

Problem (3) is worth 12 points.

Problem (4) is worth 12 points.

Problem (5) is worth 12 points.

Problem (6) is worth 24 points.

Problem (7) is worth 16 points.

Problem (8) is worth 12 points.

Problem (9) is worth 18 points.

Problem (10) is worth 16 points.

PART II

Problem (1) is worth 30 points.

Problem (2) is worth 30 points.

Problem (3) is worth 30 points.

Problem (4) is worth 30 points.

Problem (5) is worth 30 points.

PART I.

(1) Let $P = (11, 3, 8)$ and $Q = (2, 9, 6)$ for each part of this problem.

(a) Calculate the vector $\overrightarrow{PQ}$.

$\overrightarrow{PQ} = \boxed{}$

(b) Calculate the distance from P to Q.

Distance $= \boxed{}$

(c) Write the parametric equations for the line passing through P and Q.

(2) Let $\vec{u} = \langle 1, -2, 2 \rangle$ and $\vec{v} = \langle -1, 0, -1 \rangle$. Calculate the smallest angle θ between these two vectors. Simplify your answer so that it does not involve any inverse trigonometric functions.

$\theta =$ [ ]

(3) Calculate the area of the triangle with vertices $(1, 3, 1)$, $(1, 5, 4)$, and $(2, 3, -1)$.

Area $=$ [ ]

(4) (a) If the Cartesian coordinates (rectangular coordinates) of a point are $(0, \sqrt{3}, -1)$, then what are its spherical coordinates? Simplify your answer so that it does not involve any inverse trigonometric functions.

$(\rho, \theta, \phi) =$ [ ]

(b) If the spherical coordinates of a point are $(\rho, \theta, \phi) = (3, \pi, \pi/2)$, then what are its Cartesian coordinates? Simplify your answer so that it does not involve any trigonometric functions.

$(x, y, z) =$ [ ]

(5) Calculate

$$\lim_{h \to 0} \frac{e^{x(y+h)} - e^{xy}}{h}.$$

Answer: [ ]

(6) Calculate the following integrals.

(a) $\int_0^1 \int_0^x x^3 y \, dy \, dx$

Answer:

(b) $\iint_R dA$ where R is the region in the first quadrant bounded by the circle $x^2 + y^2 = 1$ and the coordinate axes (You do not need to show work on this problem.)

Answer:

(b) $\int_0^\pi \int_0^{\pi/2} \int_0^2 \rho \cos\theta \, d\rho \, d\theta \, d\phi$

Answer:

(7) Let $f(x, y) = 3x^2y + 3x^2 - y^3$. Determine whether $(1, -1)$ is the location of a local maximum, a local minimum, a saddle point, or not a critical point. (Your final answer should be one of these four possibilities.) Be sure to justify your answer.

Answer:

(8) Determine the equation of the tangent plane to the surface $z = x^2 - 2y^2$ at the point $(3, 2, 1)$.

Equation of Plane:

(9) Let

$$f(x, y) = 5x^2 + 2xy + 2y^2 - 4x - 2y + 7.$$

Determine the absolute maximum value and absolute minimum value of $f(x, y)$ on the square $\{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq 1\}$. Be sure to completely justify your answers.

Absolute Maximum Value:

Absolute Minimum Value:

(10) Calculate $\int_{\mathcal{C}} 2y\,dx + x\,dy + 2(y-z)\,dz$ where $\mathcal{C}$ is the curve given by $x = 1-t$, $y = t^2$, and $z = t^2 - t$ from $t = 0$ to $t = 1$.

Answer:

PART II. Answer each of the following. Make sure your work is clear. If you do not know how to answer a problem, tell me what you know that you think is relevant to the problem. If you end up with an answer that you think is incorrect, tell me this as well. Better yet, tell me why you think it is incorrect. In other words, let me know what you know.

(1) (a) Explain why the line given by the parametric equations $x = 2 - t$, $y = t$, and $z = 2 + t$ is on the plane $3x - y + 4z = 14$.

(b) The line in part (a) is on a plane $\mathcal{P}$ that is perpendicular to the plane in part (a). What is the equation of plane $\mathcal{P}$?

Equation of Plane: | |
|---|

(2) Using Lagrange multipliers, determine the absolute maximum value and absolute minimum value of $f(x, y) = 3x^2y + 3x^3 + 2y^3$ given the constraint $x^2 + y^2 = 1$. (You must make appropriate use of Lagrange multipliers to receive credit for this problem.)

Absolute Maximum Value:

Absolute Minimum Value:

(3) Calculate $\iiint_S (x^2+y^2)^{3/2}\,dV$ where S is the solid inside the cylinder $x^2+y^2=1$, below the cone $z=4-\sqrt{x^2+y^2}$ and above the plane $z=0$.

Answer: $\boxed{}$

(4) Using Green's Theorem, calculate the line integral

$$\int_{\mathcal{C}} (y^2 + x^3 - 2x)\, dx \; + \; (2xy + x^2 - 3\cos(y^2+1))\, dy,$$

where $\mathcal{C}$ is the triangle oriented counter-clockwise with vertices at $(0, 0)$, $(2, 2)$, and $(0, 1)$. (You must make appropriate use of Green's Theorem to receive credit for this problem.)

Answer:

(5) Calculate

$$\int_0^\pi \int_0^2 \int_0^1 zx^2 \sin(xyz)\, dx\, dy\, dz.$$

(Hint: If you do this problem the way I intend, then the integration should not be hard.)

Answer: [ ]