Test 3 Review

Problem 1 (c) (1999):

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$$\int_{-1}^{1}\int_{y^2}^{1}\cos(x^{3/2})\,dx\,dy = \;?$$

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$$-1 \leq y \leq 1$$

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$$\int_{-1}^{1}\int_{y^2}^{1}\cos(x^{3/2})\,dx\,dy = \ ?$$

$$-1 \le y \le 1$$
$$y^2 \le x \le 1$$

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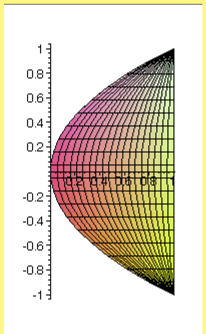

$$-1 \le y \le 1$$
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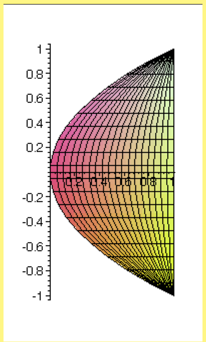

$$-1 \le y \le 1$$
$$y^2 \le x \le 1$$

$$\int\;\int\qquad\cos(x^{3/2})\,dy\,dx = \;?$$

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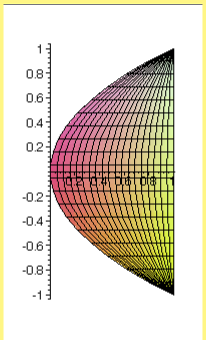

$$-1 \leq y \leq 1$$
$$y^2 \leq x \leq 1$$

$$\int_{0}^{1}\int \qquad \cos(x^{3/2})\,dy\,dx = \ ?$$

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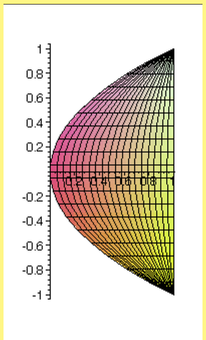

$$-1 \le y \le 1$$
$$y^2 \le x \le 1$$

$$\int_{0}^{1}\int_{-\sqrt{x}}^{\sqrt{x}}\cos(x^{3/2})\,dy\,dx = \ ?$$

$$\int_0^1 \int_{-\sqrt{x}}^{\sqrt{x}} \cos(x^{3/2})\, dy\, dx$$

$$\int_0^1 \int_{-\sqrt{x}}^{\sqrt{x}} \cos(x^{3/2})\, dy\, dx$$

$$= \int_0^1 2\sqrt{x}\cos(x^{3/2})\, dx$$

$$\int_0^1 \int_{-\sqrt{x}}^{\sqrt{x}} \cos(x^{3/2})\, dy\, dx$$

$$= \int_0^1 2\sqrt{x} \cos(x^{3/2})\, dx$$

$$= \frac{4}{3} \sin(x^{3/2}) \Big|_0^1$$

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$$= \int_0^1 2\sqrt{x}\cos(x^{3/2})\, dx$$

$$= \frac{4}{3}\sin(x^{3/2})\Big|_0^1$$

$$= \frac{4}{3}\sin(1)$$

$$\int_0^1 \int_{-\sqrt{x}}^{\sqrt{x}} \cos(x^{3/2})\, dy\, dx$$

$$= \int_0^1 2\sqrt{x}\cos(x^{3/2})\, dx$$

$$= \frac{4}{3}\sin(x^{3/2})\Big|_0^1$$

$$= \boxed{\frac{4}{3}\sin(1)}$$

Problem 3 (1999):

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$$f(x,y) = \begin{cases} 3 & \text{if } 0 \leq x \leq 2 \text{ and } 0 \leq y \leq 2 \\ -1 & \text{if } 0 \leq x \leq 2 \text{ and } 2 < y \leq 3 \\ 2 & \text{if } 2 < x \leq 3 \text{ and } 0 \leq y \leq 3. \end{cases}$$

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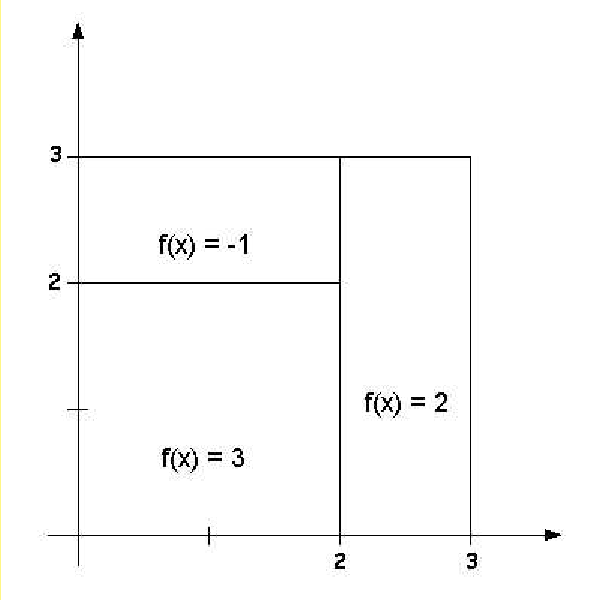
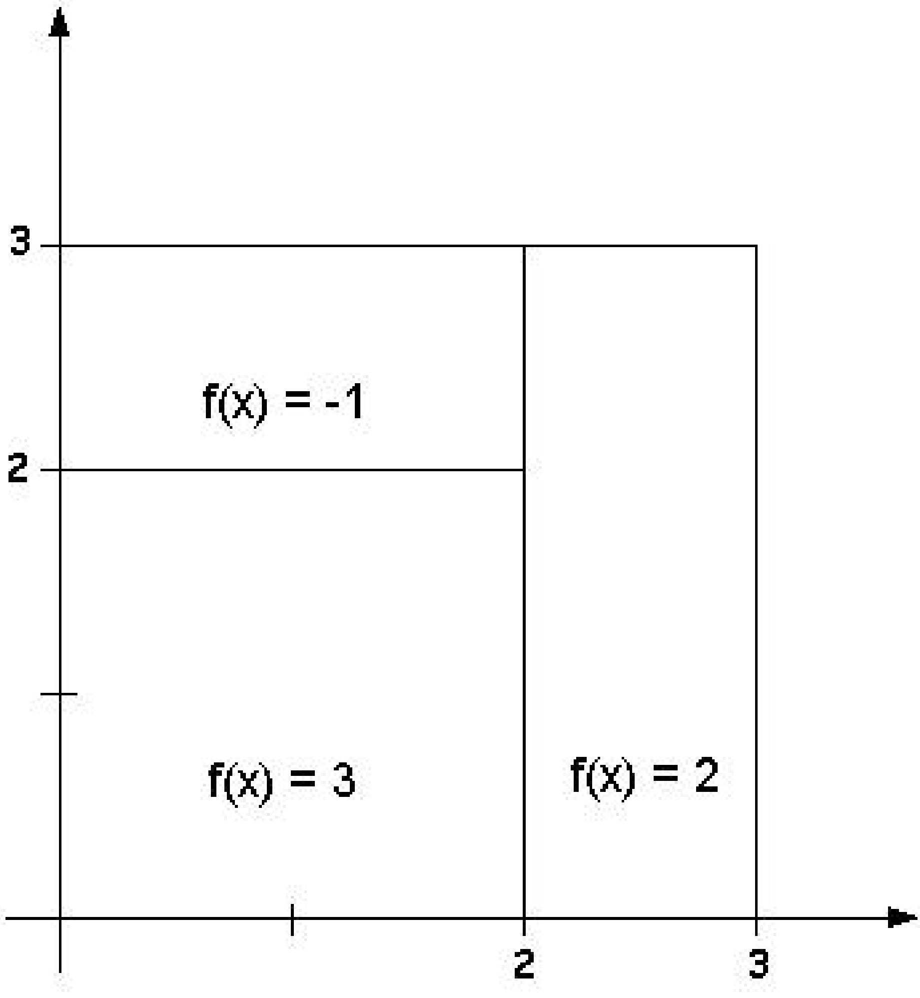
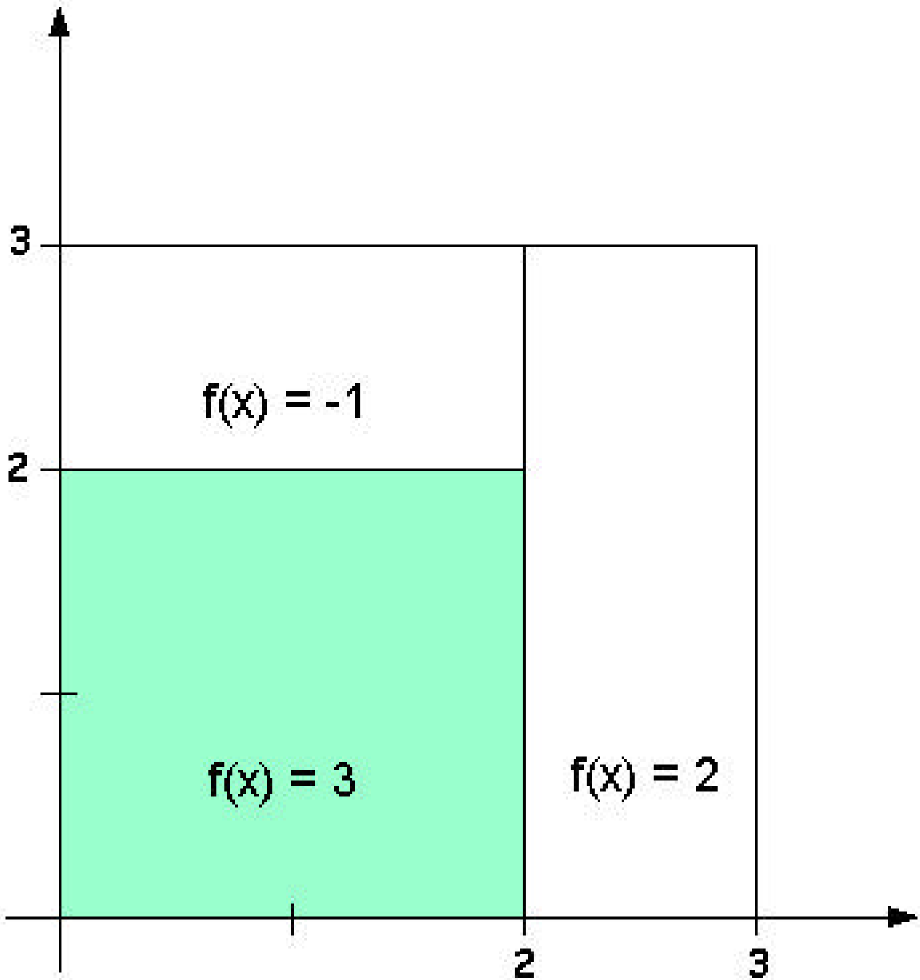
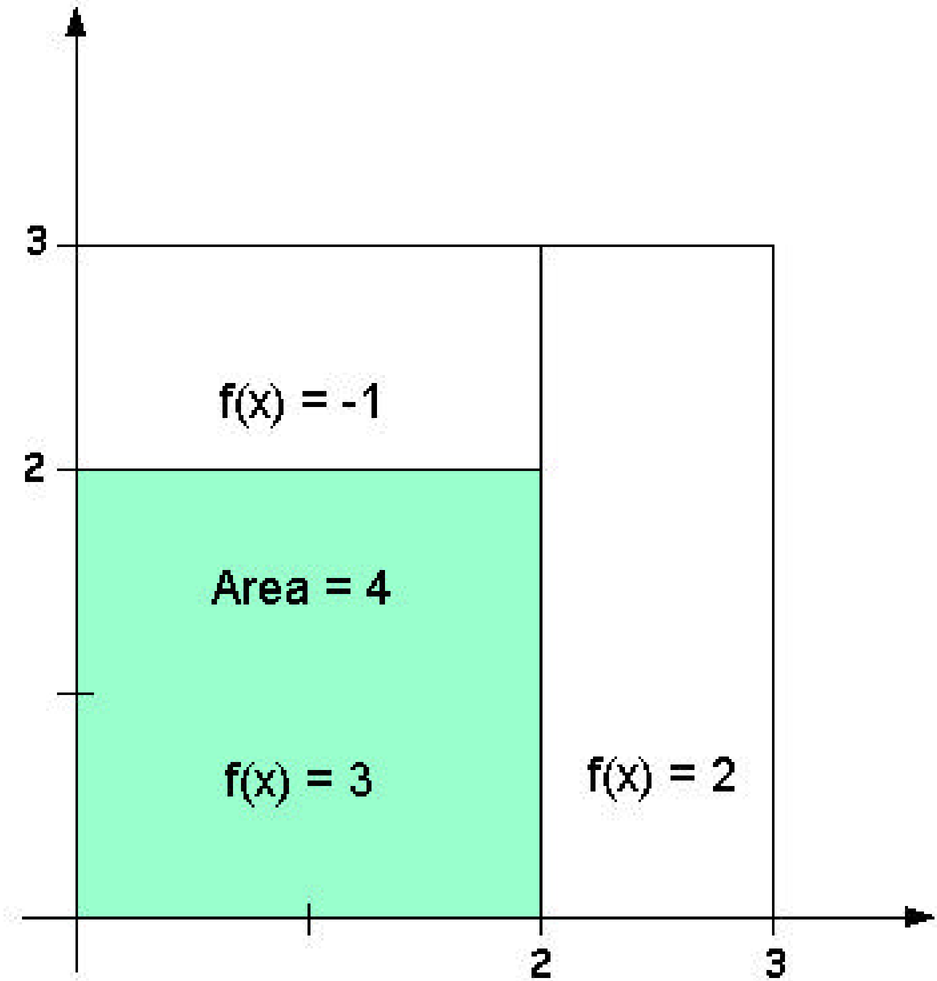
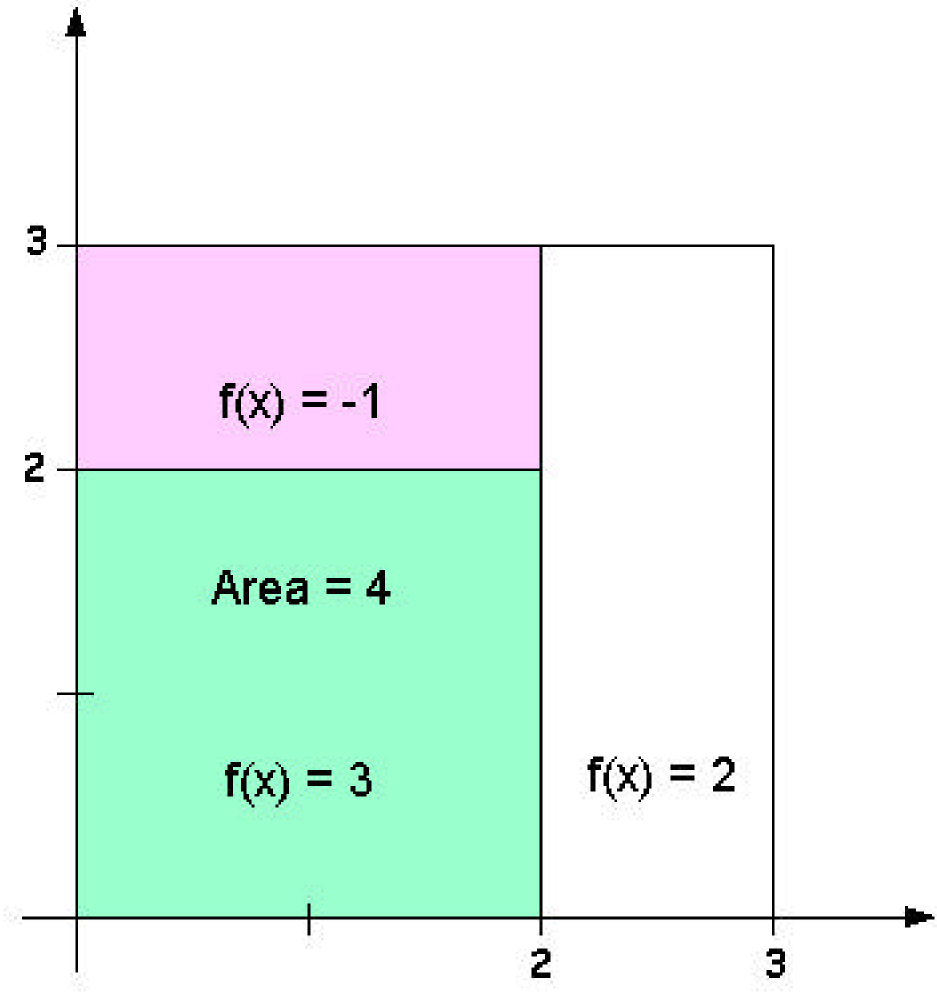


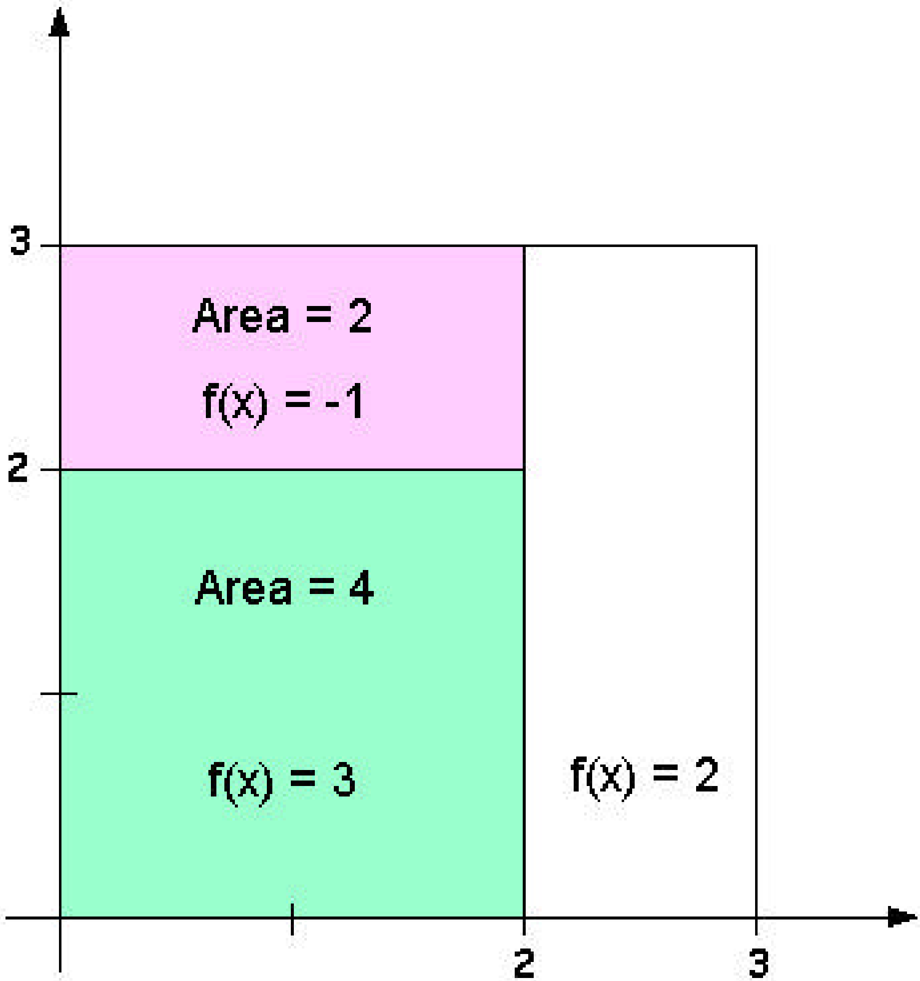


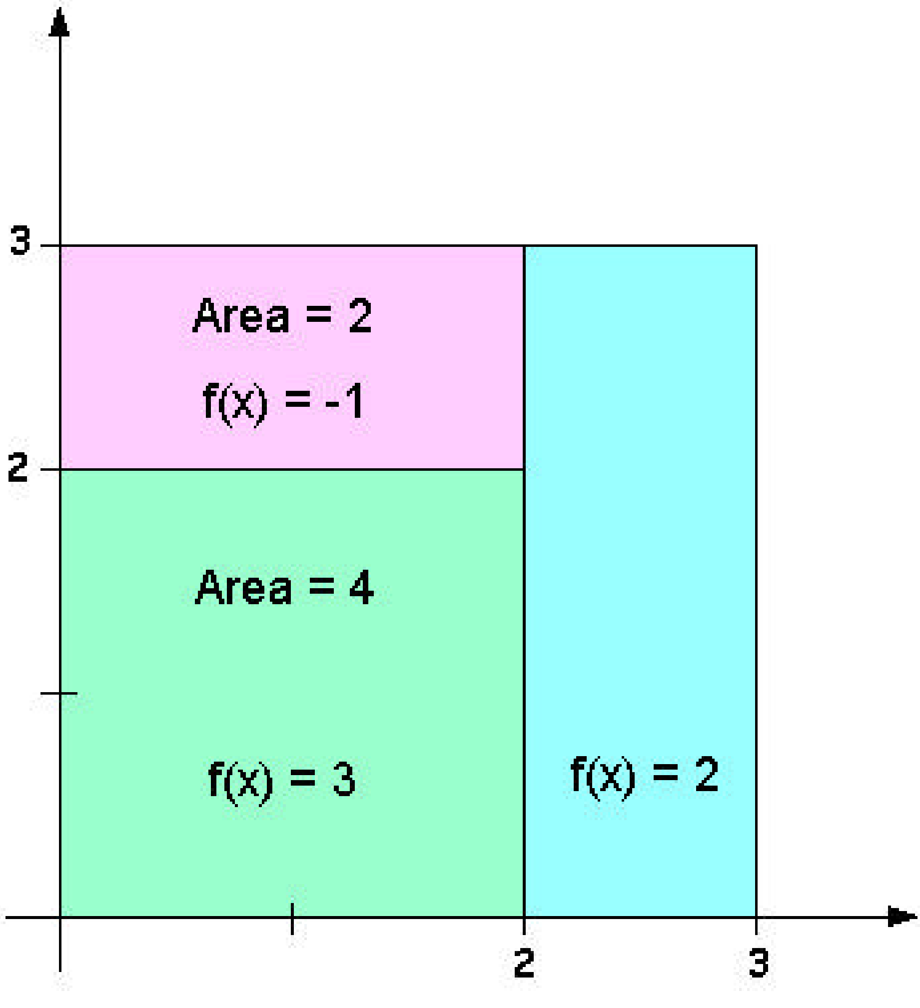
3
2
f(x) = -1
f(x) = 3
f(x) = 2
2
3

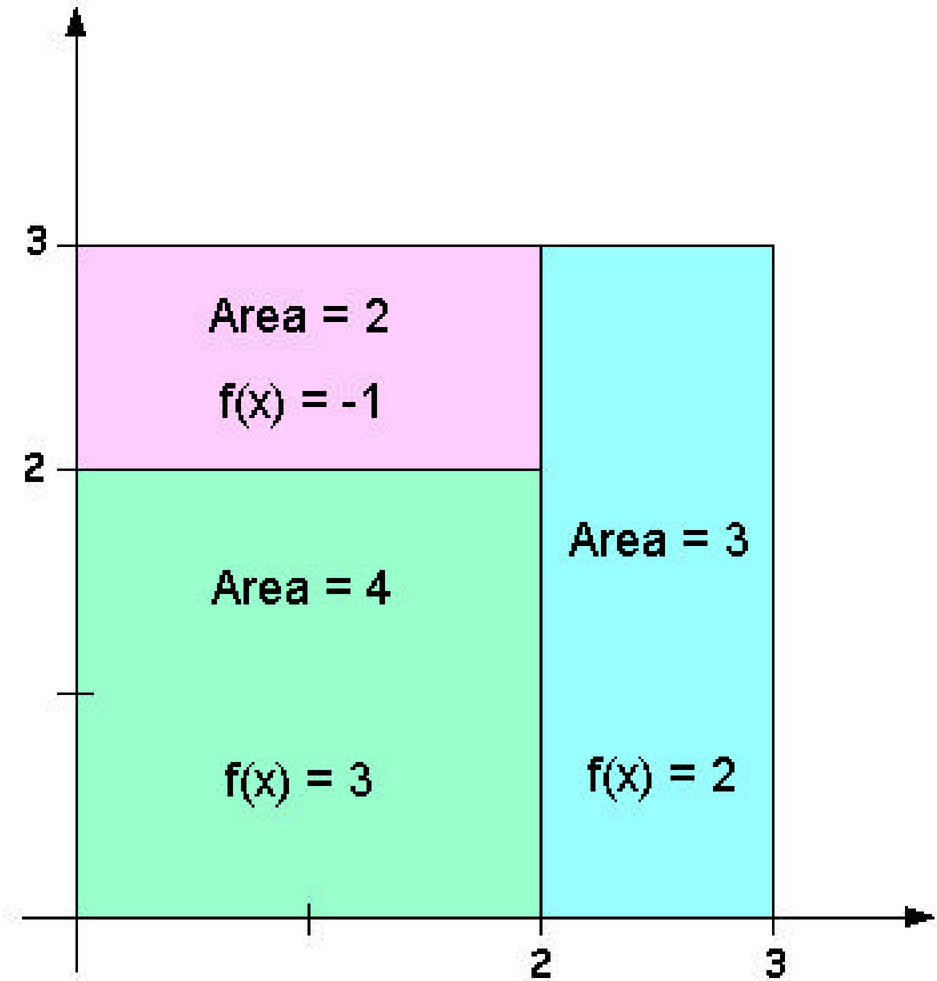

3
f(x) = -1
2
f(x) = 3
f(x) = 2
2
3

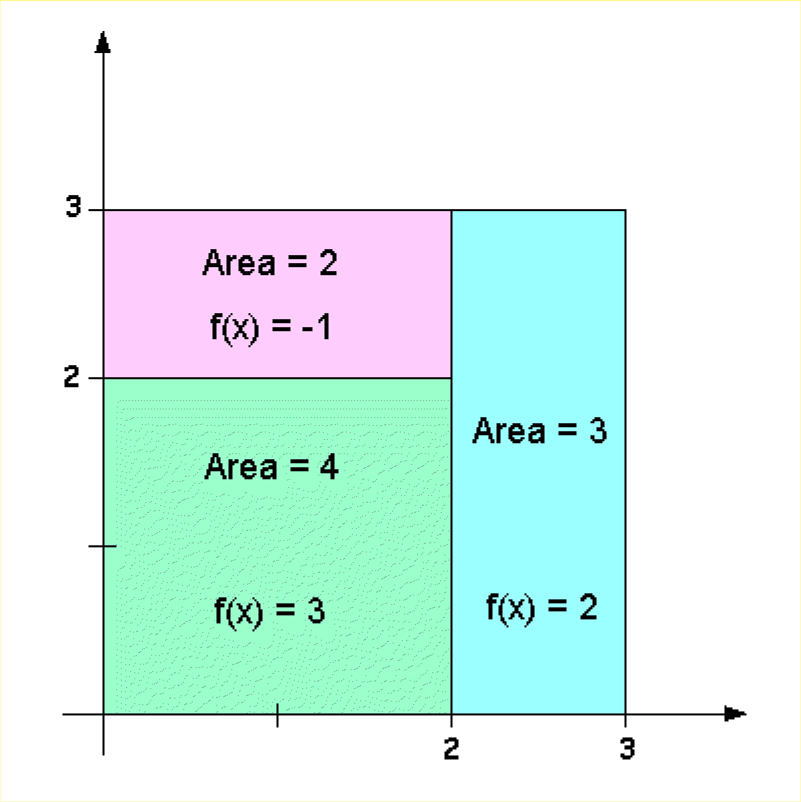

3
2
f(x) = -1
Area = 4
f(x) = 3
f(x) = 2
2
3


3
f(x) = -1
2
Area = 4
f(x) = 3
f(x) = 2
2
3

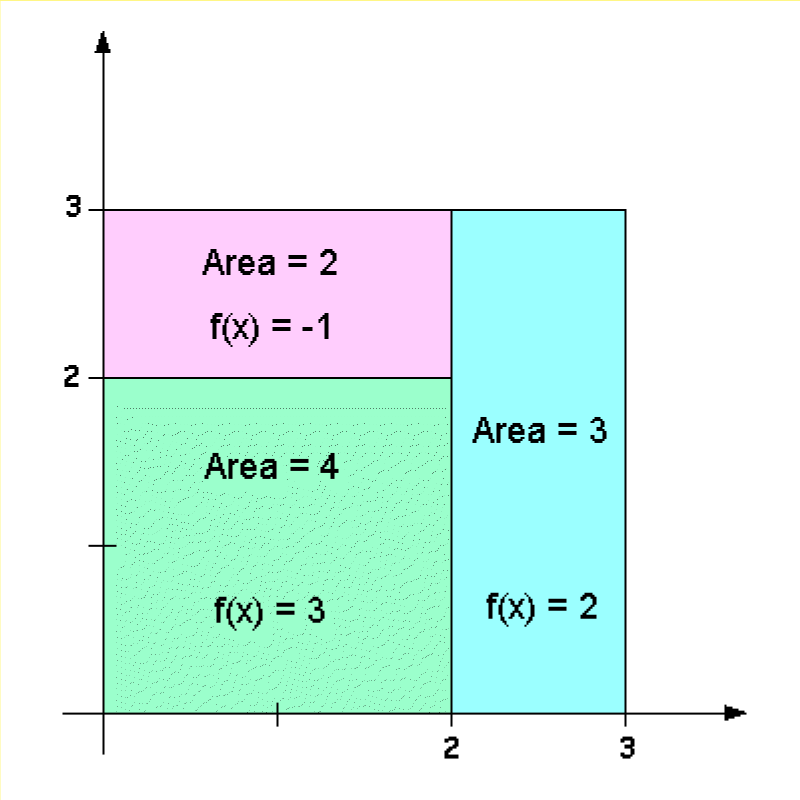

3
Area = 2
f(x) = -1
2
Area = 4
f(x) = 3
f(x) = 2
2
3


3
Area = 2
f(x) = -1
2
Area = 4
f(x) = 3
f(x) = 2
2
3

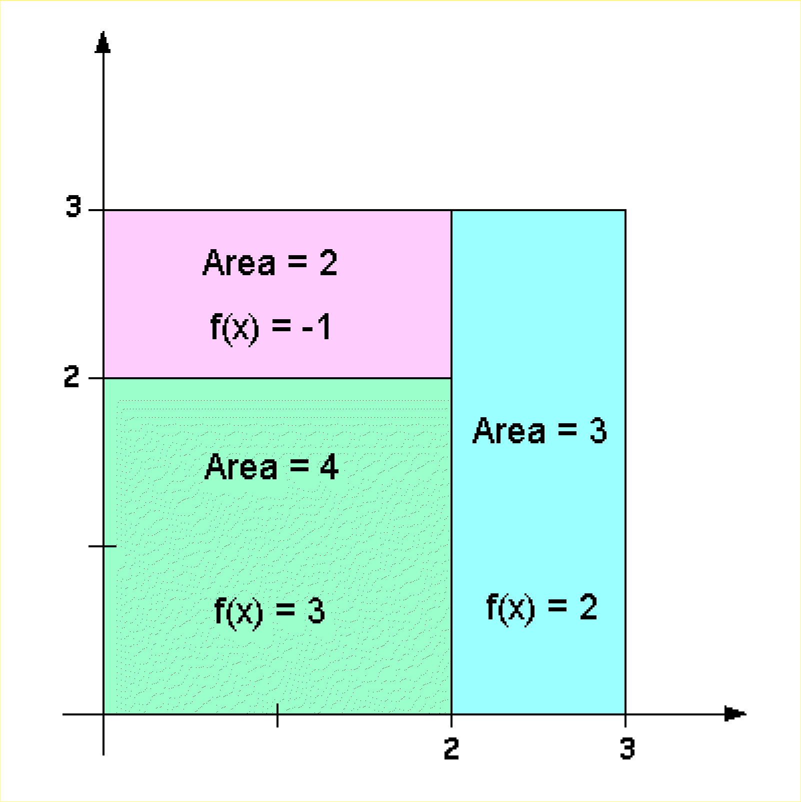

3
Area = 2
f(x) = -1
2
Area = 3
Area = 4
f(x) = 3
f(x) = 2
2
3



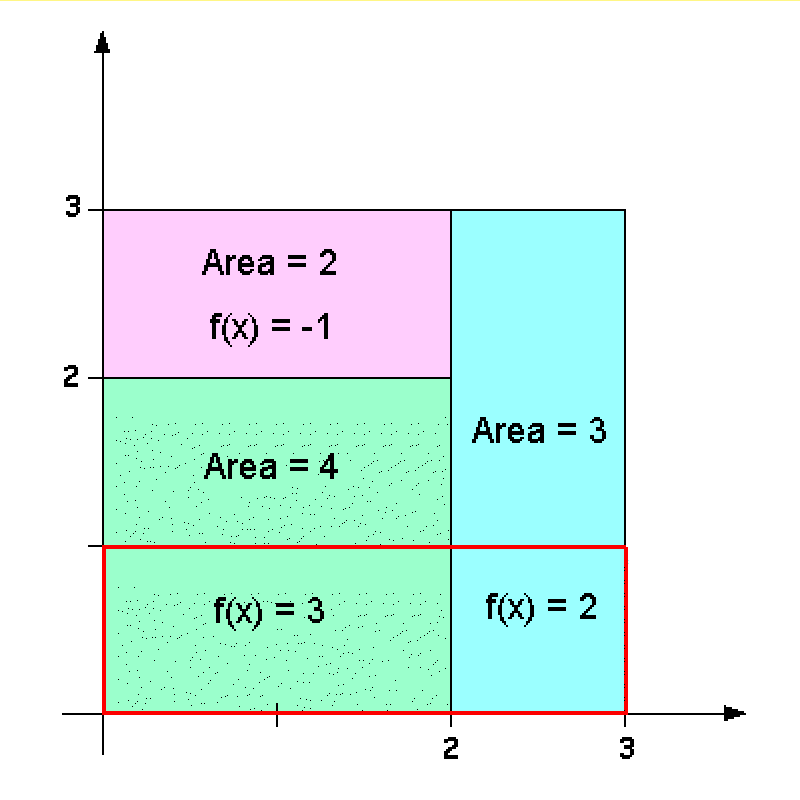
$$\iint_R f(x,y)\,dA =$$



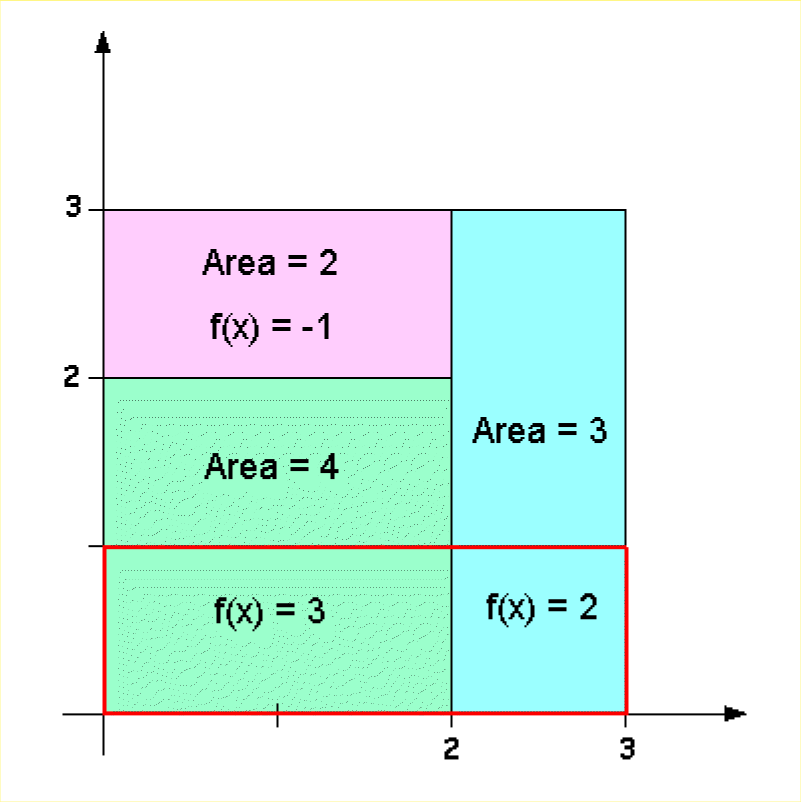
$$\iint_R f(x,y)\,dA = 4 \times 3 + 2 \times (-1) + 3 \times 2 = 16$$



$$\int_0^3 \int_0^1 f(x, y)\, dy\, dx =$$



$$\int_0^3 \int_0^1 f(x,y)\,dy\,dx =$$



$$\int_0^3 \int_0^1 f(x,y)\,dy\,dx = 2 \times 3 + 1 \times 2 = 8$$

Problem 4 (1999):

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$$(x, y, z) = (\sqrt{2}, \sqrt{2}, 2)$$

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Calculate (r, θ, z) and (ρ, θ, ϕ).

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$$r^2 =$$

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$$(x, y, z) = (\sqrt{2}, \sqrt{2}, 2)$$

Calculate (r, θ, z) and (ρ, θ, ϕ).

$$r^2 = x^2 + y^2 = 4$$

Problem 4 (1999):

$$(x, y, z) = (\sqrt{2}, \sqrt{2}, 2)$$

Calculate (r, θ, z) and (ρ, θ, ϕ).

$$r^2 = x^2 + y^2 = 4 \quad \text{so} \quad r = 2$$

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$$(x, y, z) = (\sqrt{2}, \sqrt{2}, 2)$$

Calculate (r, θ, z) and (ρ, θ, ϕ).

$$r^2 = x^2 + y^2 = 4 \quad \text{so} \quad r = 2$$
$$x = y > 0$$

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$$r^2 = x^2 + y^2 = 4 \quad \text{so} \quad r = 2$$
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$$(r, \theta, z) = (2, \pi/4, 2)$$

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$$\rho^2 = x^2 + y^2 + z^2 = 8$$

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$$z =$$

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$$z = \rho \cos \phi,$$

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$$\theta = \theta \quad \text{so} \quad \theta = \pi/4$$

$$z = \rho \cos\phi, \; \cos\phi = 1/\sqrt{2}$$

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$$\theta = \theta \quad \text{so} \quad \theta = \pi/4$$
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Problem 6 (1999):

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Express the volume of the solid above the surface $z = x^2 + y^2$ and below the surface $x^2 + y^2 + z^2 = 6$ as an iterated integral in polar or cylindrical coordinates. Do not evaluate the integral.

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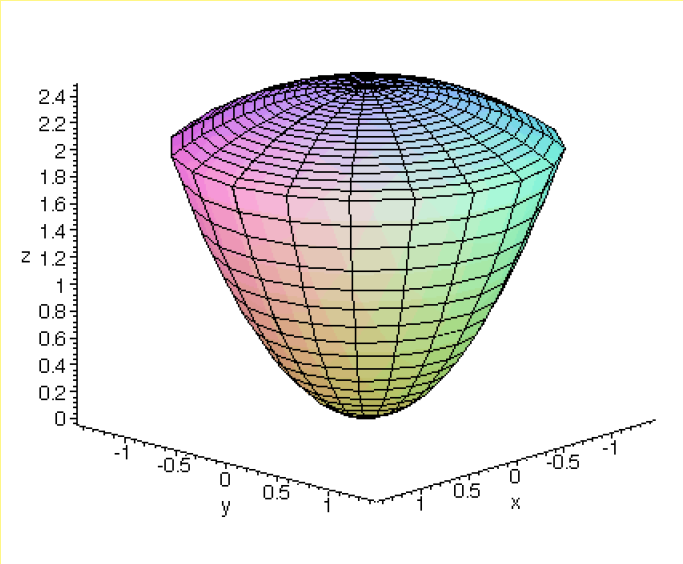

$$z = x^2 + y^2 \qquad x^2 + y^2 + z^2 = 6$$

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$$z + z^2 = 6$$

$$z = x^2 + y^2 \qquad x^2 + y^2 + z^2 = 6$$

$$z + z^2 = 6$$
$$z^2 + z - 6 = 0$$

$$z = x^2 + y^2 \qquad x^2 + y^2 + z^2 = 6$$

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$$(z+3)(z-2) = 0$$

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$$(z+3)(z-2) = 0$$

$$z = 2$$

$$z = x^2 + y^2 \qquad x^2 + y^2 + z^2 = 6$$

$$z + z^2 = 6$$
$$z^2 + z - 6 = 0$$
$$(z+3)(z-2) = 0$$
$$z = 2$$

The surfaces intersect on the plane $z = 2$.

$$z = x^2 + y^2 \qquad x^2 + y^2 + z^2 = 6$$

$$z + z^2 = 6$$

$$z^2 + z - 6 = 0$$

$$(z+3)(z-2) = 0$$

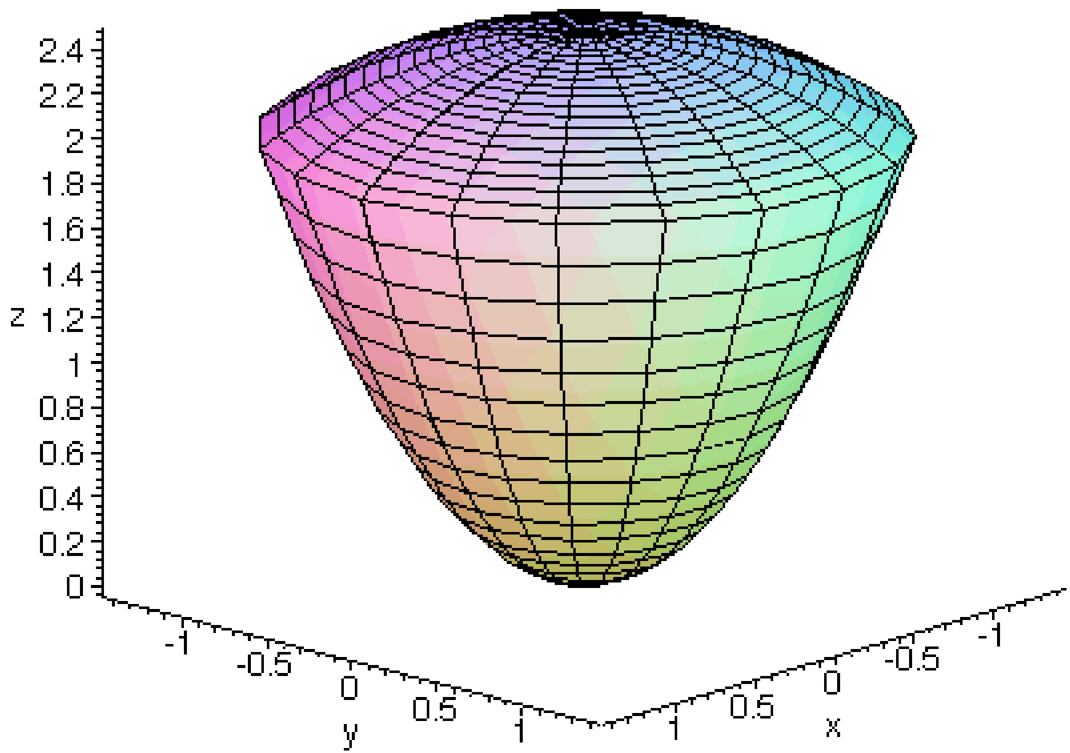
$$z = 2$$

The surfaces intersect on the plane $z = 2$.
The projection of this intersection to the xy-plane is

$$z = x^2 + y^2 \qquad\qquad x^2 + y^2 + z^2 = 6$$

$$z + z^2 = 6$$
$$z^2 + z - 6 = 0$$
$$(z+3)(z-2) = 0$$
$$z = 2$$

The surfaces intersect on the plane $z = 2$.
The projection of this intersection to the xy-plane is the circle $x^2 + y^2 = 2$.



z
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-1
-0.5
0
0.5
1
y
1
0.5
0
-0.5
-1
x

$$z = x^2 + y^2 \qquad x^2 + y^2 + z^2 = 6$$

The projection of this intersection to the xy-plane is the circle $x^2 + y^2 = 2$.

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Express the volume using polar or cylindrical coordinates.

$$z = x^2 + y^2 \qquad x^2 + y^2 + z^2 = 6$$

The projection of this intersection to the xy-plane is the circle $x^2 + y^2 = 2$.

Express the volume using polar or cylindrical coordinates.

$$\int \quad \int \quad \int \qquad dz\, dr\, d\theta$$

$$z = x^2 + y^2 \qquad x^2 + y^2 + z^2 = 6$$

The projection of this intersection to the xy-plane is the circle $x^2 + y^2 = 2$.

Express the volume using polar or cylindrical coordinates.

$$\int \quad \int \quad \int \qquad r \, dz \, dr \, d\theta$$

$$z = x^2 + y^2 \qquad x^2 + y^2 + z^2 = 6$$

The projection of this intersection to the xy-plane is the circle $x^2 + y^2 = 2$.

Express the volume using polar or cylindrical coordinates.

$$\int_0^{2\pi} \int \quad \int \qquad r \, dz \, dr \, d\theta$$

$$z = x^2 + y^2 \qquad x^2 + y^2 + z^2 = 6$$

The projection of this intersection to the xy-plane is the circle $x^2 + y^2 = 2$.

Express the volume using polar or cylindrical coordinates.

$$\int_0^{2\pi} \int_0^{\sqrt{2}} \int \qquad r \, dz \, dr \, d\theta$$

$$z = x^2 + y^2 \qquad x^2 + y^2 + z^2 = 6$$

The projection of this intersection to the xy-plane is the circle $x^2 + y^2 = 2$.

Express the volume using polar or cylindrical coordinates.

$$\int_0^{2\pi} \int_0^{\sqrt{2}} \int_{r^2}^{\sqrt{6-r^2}} r \, dz \, dr \, d\theta$$

$$z = x^2 + y^2 \qquad x^2 + y^2 + z^2 = 6$$

The projection of this intersection to the xy-plane is the circle $x^2 + y^2 = 2$.

Express the volume using polar or cylindrical coordinates.

$$\boxed{\int_0^{2\pi} \int_0^{\sqrt{2}} \int_{r^2}^{\sqrt{6-r^2}} r \, dz \, dr \, d\theta}$$

Problem 7 (1999):

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Using spherical coordinates, calculate

$$\iiint_S \left(x^2 + y^2 + z^2\right)^{3/2} dV$$

where S is the solid bounded above by the sphere $x^2 + y^2 + z^2 = 1$ and below by the plane $z = 0$.

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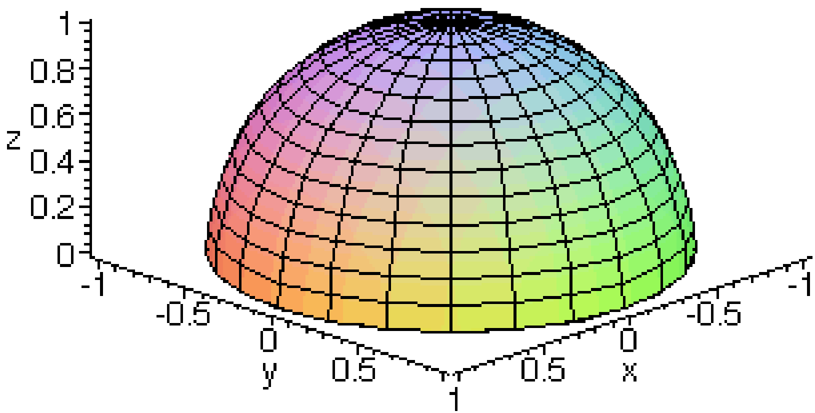

1
0.8
0.6
z
0.4
0.2
0
-1
-0.5
0
y
0.5
1
0.5
0
x
-0.5
-1

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$$\int \quad \int \quad \int \qquad\qquad\qquad d\rho\, d\theta\, d\phi$$

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where S is the solid bounded above by the sphere $x^2 + y^2 + z^2 = 1$ and below by the plane $z = 0$.

$$\int \quad \int \quad \int \qquad\qquad \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$$

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$$\int \quad \int \quad \int (\rho^2)^{3/2} \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$$

Problem 7 (1999):

Using spherical coordinates, calculate

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where S is the solid bounded above by the sphere $x^2 + y^2 + z^2 = 1$ and below by the plane $z = 0$.

$$\int_0^{\pi/2} \int \quad \int (\rho^2)^{3/2}\, \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$$

Problem 7 (1999):

Using spherical coordinates, calculate

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$$
\begin{aligned}
\int_0^{\pi/2}\int_0^{2\pi}\int_0^1 (\rho^2)^{3/2}\,\rho^2 \sin\phi \, d\rho \, d\theta \, d\phi \\
&= \int_0^{\pi/2}\int_0^{2\pi}\int_0^1 \rho^5 \sin\phi \, d\rho \, d\theta \, d\phi \\
&= \frac{1}{6}\int_0^{\pi/2}\int_0^{2\pi} \sin\phi \, d\theta \, d\phi \\
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\end{aligned}
$$

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\end{aligned}
$$

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\int_0^{\pi/2} \int_0^{2\pi} \int_0^1 (\rho^2)^{3/2}\, \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi
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&= \frac{1}{6} \int_0^{\pi/2} \int_0^{2\pi} \sin\phi \, d\theta \, d\phi \\
&= \frac{\pi}{3} \int_0^{\pi/2} \sin\phi \, d\phi \\
&= \frac{\pi}{3} (-\cos\phi) \Big|_0^{\pi/2} = \boxed{\frac{\pi}{3}}
\end{aligned}$$

Other Answers From 1999:

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Problem (1)(a): $1/2$

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Problem (5): $\int_0^3 \int_0^2 \int_0^{\sqrt{4-y^2}} dz\, dy\, dx$

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- For polar coordinates, remember $dA = r\,dr\,d\theta$.
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$$dV = r\,dz\,dr\,d\theta.$$

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- I will practice problems like Problem (3) on this test.