Test 2 Review

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$$f(x, y) = x^2y^2 + x + y$$

(b) There are infinitely many different values for the directional derivative of $f(x, y)$ at the point $(2, 1)$ (since there are infinitely many directions that can be used to compute the directional derivative). Which of these is maximal? In other words, what is the largest value of the directional derivative of $f(x, y)$ at the point $(2, 1)$?

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$$|\nabla f(2, 1)|$$

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$$|\nabla f(2, 1)| = |\langle 5, 9\rangle| = \sqrt{25 + 81} = \boxed{\sqrt{106}}$$

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Using the Chain Rule, compute $\dfrac{\partial w}{\partial t}$ where

$$w = x^2 + xyz + x + 2z, \; x = t\sin(\sqrt{s}) + 2^s - t^2$$

$$y = 2s + s^2\sin(t), \text{ and } z = t^2 s^3 - 2t$$

You do not need to put your answer in terms of s and t (the variables x, y, and z can appear in your answer).

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$$= (2x + yz + 1)$$

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$$= (2x + yz + 1)(\sin(\sqrt{s}) - 2t)$$

$$+ xz(s^2\cos t) + (xy + 2)(2ts^3 - 2)$$

Problem 6 (1999):

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Find the critical points of the function

$$f(x, y) = 3x + xy^2$$

where (x, y) is restricted to points in the set

$$S = \{(x, y) : x^2 + y^2 \leq 9\}.$$

Also, determine the maximum and the minimum values of $f(x, y)$ in S as well as all points (x, y) where these extreme values occur.

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STOP!!

$$f(x,y) = 3x + xy^2, \quad S = \{(x,y) : x^2 + y^2 \leq 9\}$$

$$f_x = 3 + y^2$$

DON'T COMPUTE f_y!!

$$f(x,y) = 3x + xy^2, \quad S = \{(x,y) : x^2 + y^2 \leq 9\}$$

$$f_x = 3 + y^2$$

f_x NEVER EQUALS ZERO!!

$$f(x,y) = 3x + xy^2, \quad S = \{(x,y) : x^2 + y^2 \leq 9\}$$

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THE CRITICAL POINTS ARE
THE POINTS ON THE BOUNDARY!!

$$f(x,y) = 3x + xy^2, \quad S = \{(x,y) : x^2 + y^2 \leq 9\}$$

$$f_x = 3 + y^2$$

Critical Points: all points (x, y) where $x^2 + y^2 = 9$

$$f(x, y) = 3x + xy^2, \quad S = \{(x, y) : x^2 + y^2 \leq 9\}$$

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Check $x = -3$, $x = -2$, $x = 2$, and $x = 3$.

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Check $x = -3$, $x = -2$, $x = 2$, and $x = 3$.

$$g(-3) = -9, \quad g(-2) = -16, \quad g(2) = 16, \quad g(3) = 9$$

$f(x, y) = 3x + xy^2, \qquad x^2 + y^2 = 9$

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$$g(-3) = -9, \quad g(-2) = -16, \quad g(2) = 16, \quad g(3) = 9$$

The maximum is $\boxed{16}$ and it occurs at $\boxed{(2, \pm\sqrt{5})}$.

The minimum is $\boxed{-16}$ and it occurs at $\boxed{(-2, \pm\sqrt{5})}$.

Problem 7 (1999):

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Let

$$f(x,y) = x^4 + 4xy + xy^2.$$

The function $f(x,y)$ has 3 critical points. Calculate the three critical points and indicate (with justification) whether each determines a local maximum value of $f(x,y)$, a local minimum value of $f(x,y)$, or a saddle point of $f(x,y)$.

$$f(x, y) = x^4 + 4xy + xy^2$$

$$f(x, y) = x^4 + 4xy + xy^2$$

$$f_x = 4x^3 + 4y + y^2 = 0$$
$$f_y = 4x + 2xy = 0$$

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$$4x + 2xy = 0 \quad \text{implies} \quad x = 0 \text{ or } y = -2$$

If $x = 0$, then $f_x = 0$ implies $4y + y^2 = 0$ so that $y = 0$ or $y = -4$.

$$f(x,y) = x^4 + 4xy + xy^2$$

$$f_x = 4x^3 + 4y + y^2 = 0$$

$$f_y = 4x + 2xy = 0$$

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If $x = 0$, then $f_x = 0$ implies $4y + y^2 = 0$ so that $y = 0$ or $y = -4$. If $y = -2$, then $f_x = 0$ implies $4x^3 - 4 = 0$ so that $x = 1$.

$$f(x, y) = x^4 + 4xy + xy^2$$

$$f_x = 4x^3 + 4y + y^2 = 0$$
$$f_y = 4x + 2xy = 0$$

$$4x + 2xy = 0 \quad \text{implies} \quad x = 0 \text{ or } y = -2$$

If $x = 0$, then $f_x = 0$ implies $4y + y^2 = 0$ so that $y = 0$ or $y = -4$. If $y = -2$, then $f_x = 0$ implies $4x^3 - 4 = 0$ so that $x = 1$. The critical points are

$$(0, 0), \quad (0, -4), \quad \text{and} \quad (1, -2).$$

$$f(x, y) = x^4 + 4xy + xy^2$$

$$(0, 0), \quad (0, -4), \quad \text{and} \quad (1, -2).$$

$$f(x,y) = x^4 + 4xy + xy^2$$

$$(0,0), \quad (0,-4), \quad \text{and} \quad (1,-2).$$

$$f_x = 4x^3 + 4y + y^2 = 0$$

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$$f_x = 4x^3 + 4y + y^2 = 0$$

$$f_y = 4x + 2xy = 0$$

$$f_{xx} = 12x^2, \quad f_{yy} = 2x, \quad f_{xy} = 4 + 2y$$

$$f(x,y) = x^4 + 4xy + xy^2$$

$$(0,0), \quad (0,-4), \quad \text{and} \quad (1,-2).$$

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$$f_{xx} = 12x^2, \quad f_{yy} = 2x, \quad f_{xy} = 4 + 2y$$

$$D = f_{xx} f_{yy} - f_{xy}^2$$

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$$D(0,0) = 0 - 4^2 = -16 \implies$$

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$$D(0, -4) = 0 - (-4)^2 = -16 \implies \text{saddle point at } (0, -4)$$

$$D(1, -2) = 12 \cdot 2 - 0^2 = 24 \implies$$

$$f(x, y) = x^4 + 4xy + xy^2$$

$$(0, 0), \quad (0, -4), \quad \text{and} \quad (1, -2).$$

$$f_{xx} = 12x^2, \quad f_{yy} = 2x, \quad f_{xy} = 4 + 2y$$

$$D = f_{xx} f_{yy} - f_{xy}^2$$

$$D(0, 0) = 0 - 4^2 = -16 \implies \text{saddle point at } (0, 0)$$

$$D(0, -4) = 0 - (-4)^2 = -16 \implies \text{saddle point at } (0, -4)$$

$$D(1, -2) = 12 \cdot 2 - 0^2 = 24 \implies \text{local min at } (1, -2)$$

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- The gradient at a point on a surface $\boldsymbol{F = 0}$ is perpendicular to the tangent plane there.
- When taking limits, every direction counts but some directions might count more than others.
- Think polar coordinates with limits (as $\boldsymbol{(x, y) \longrightarrow (0, 0)}$).

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- $D = f_{xx}f_{yy} - f_{xy}^2$.
- I promise only to compute D at places where $\nabla f = 0$.
- If $D > 0$ and $f_{xx} > 0$, then we've located a local minimum.

Quick Overview:

- If $D > 0$ and $f_{xx} < 0$, then we've located a local maximum.

Quick Overview:

- If $D > 0$ and $f_{xx} < 0$, then we've located a local maximum.
- If $D < 0$, then we've located a saddle point.

Quick Overview:

- If $D > 0$ and $f_{xx} < 0$, then we've located a local maximum.
- If $D < 0$, then we've located a saddle point.
- I promise to look at the first two sections of Chapter 16

Quick Overview:

- If $D > 0$ and $f_{xx} < 0$, then we've located a local maximum.
- If $D < 0$, then we've located a saddle point.
- I promise to look at the first two sections of Chapter 16 (a little).

Quick Overview:

- If $D > 0$ and $f_{xx} < 0$, then we've located a local maximum.
- If $D < 0$, then we've located a saddle point.
- I promise to look at the first two sections of Chapter 16 (a little).
- I will not study Lagrange multipliers.