Test 1 Review

Problem 1 (1999):

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(b) Calculate the magnitude of vector $\overrightarrow{\boldsymbol{QP}}$.

$$\begin{aligned} |\overrightarrow{\boldsymbol{QP}}| &= |\langle \boldsymbol{2, 3, 1} \rangle| \\ &= \sqrt{\boldsymbol{2^2 + 3^2 + 1^2}} \end{aligned}$$

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(c) Calculate $\angle PQR$ and simplify your answer (it should not involve any inverse trigonometric functions).

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Let $\theta = \angle PQR$.

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$$\boldsymbol{P = (6,4,0), \quad Q = (4,1,-1), \quad R = (7,2,-3)}$$

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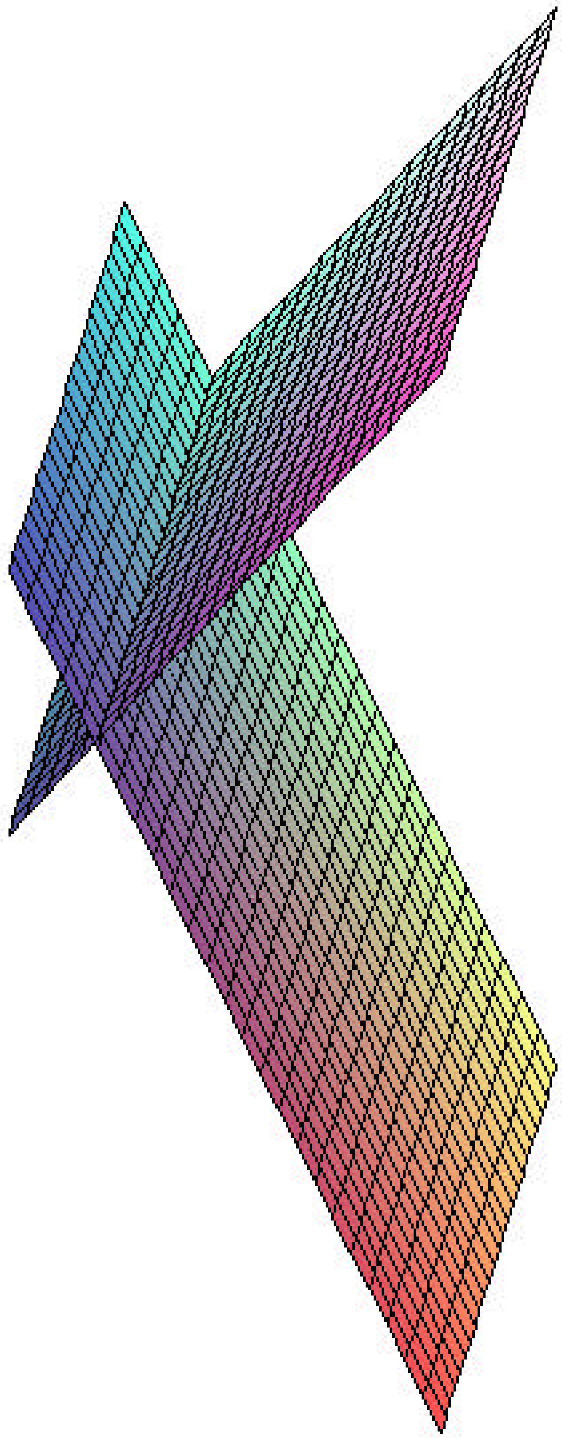

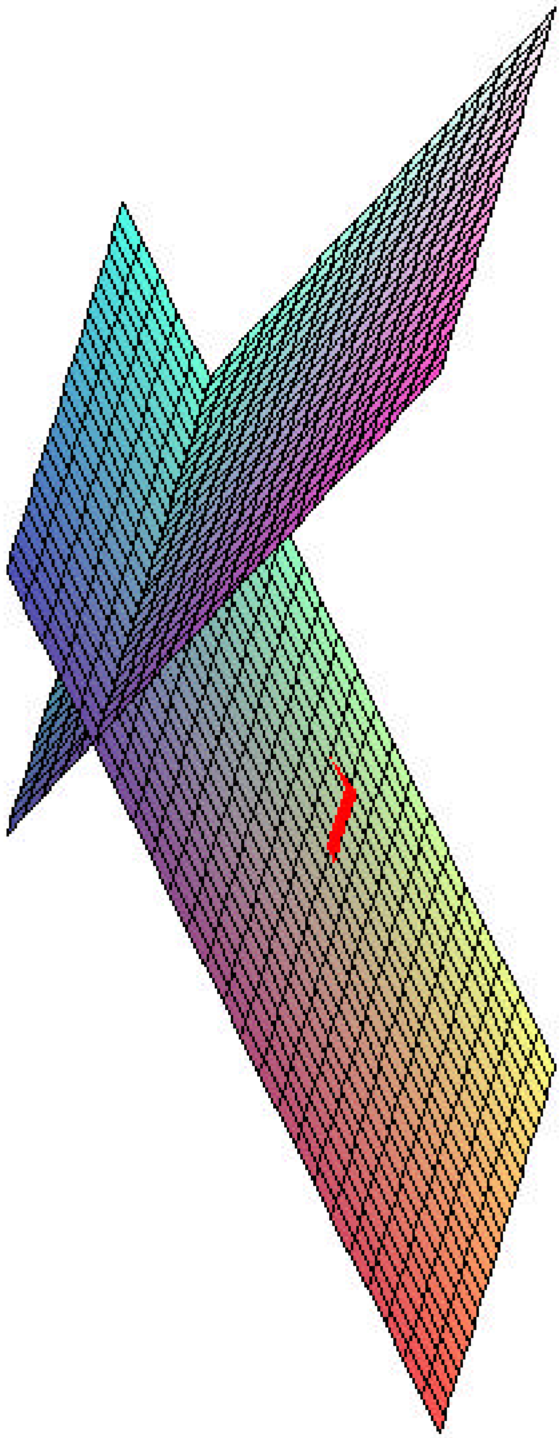

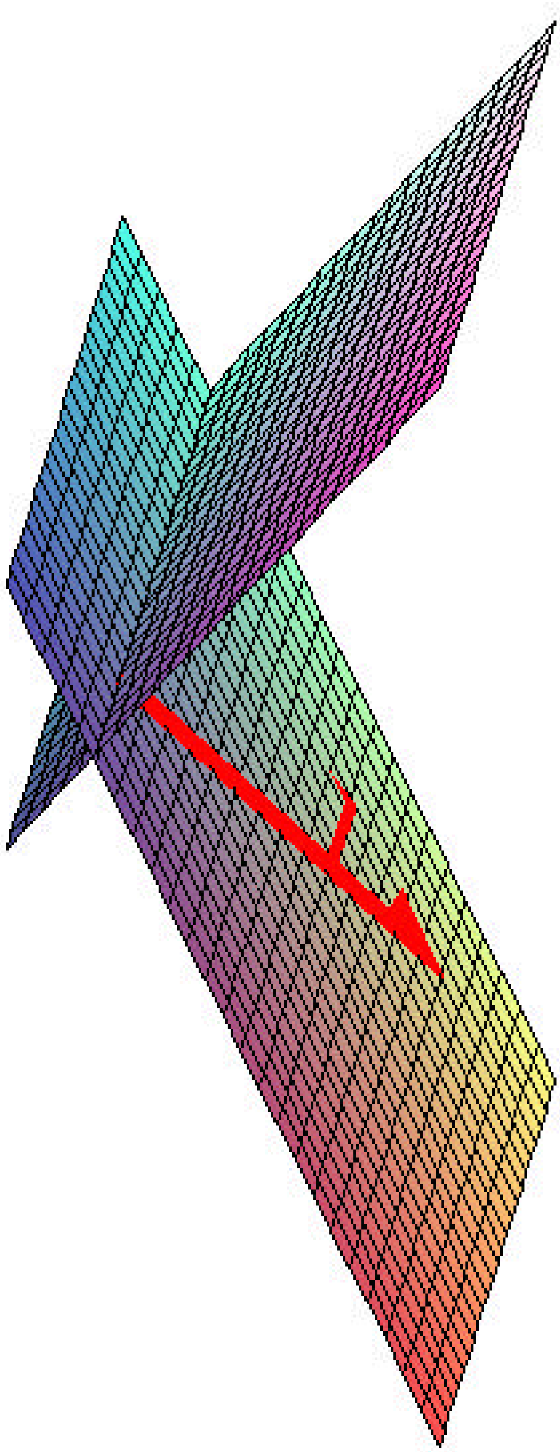

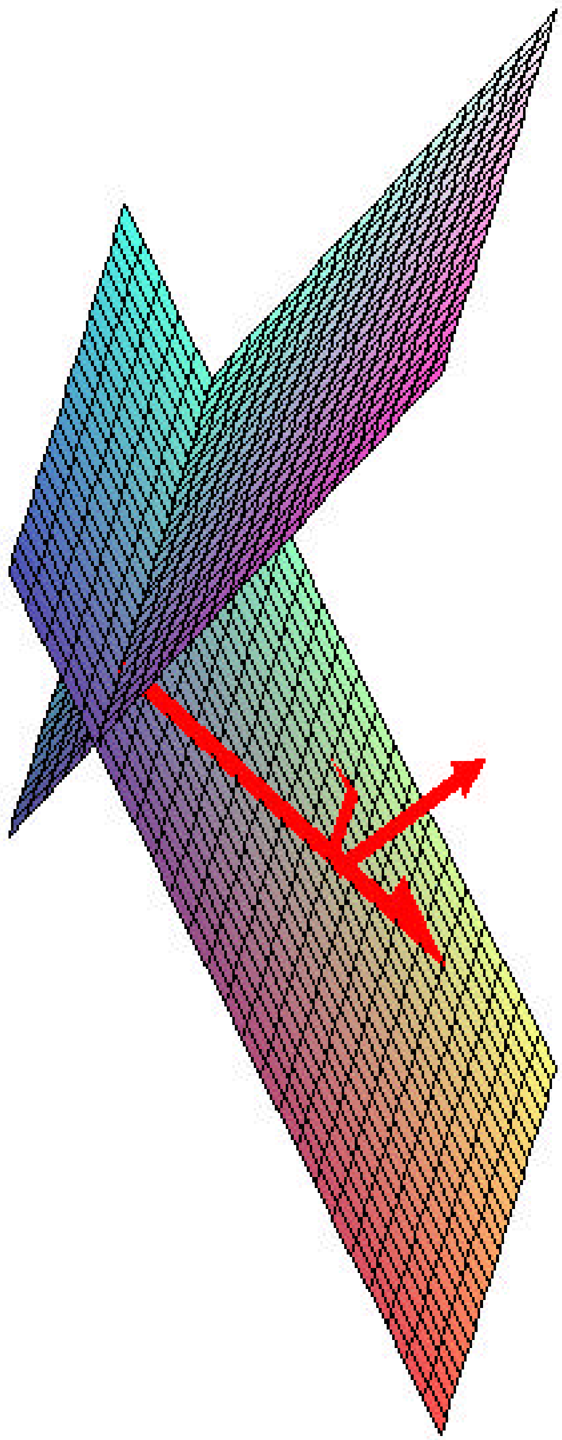

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(a) Why are these lines skew?

$$\ell_1 : \begin{cases} x = 2 + t \\ y = 0 \\ z = -1 + t \end{cases} \qquad \ell_2 : \begin{cases} x = 3 \\ y = 2t \\ z = 1 + t \end{cases}$$

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(the first component on the left is 1 and on the right is 0).

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They do not intersect since otherwise the intersection point, say P, would have x-coordinate 3 and y-coordinate 0. So P would be $(3, 0, z_0)$ for some z_0. In the equations for ℓ_1, we would have $t = 1$ so that $z_0 = 0$. In the equations for ℓ_2, we would have $t = 0$ so that

Problem 4 (1999):

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They are not parallel.

They do not intersect.

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They are not parallel.

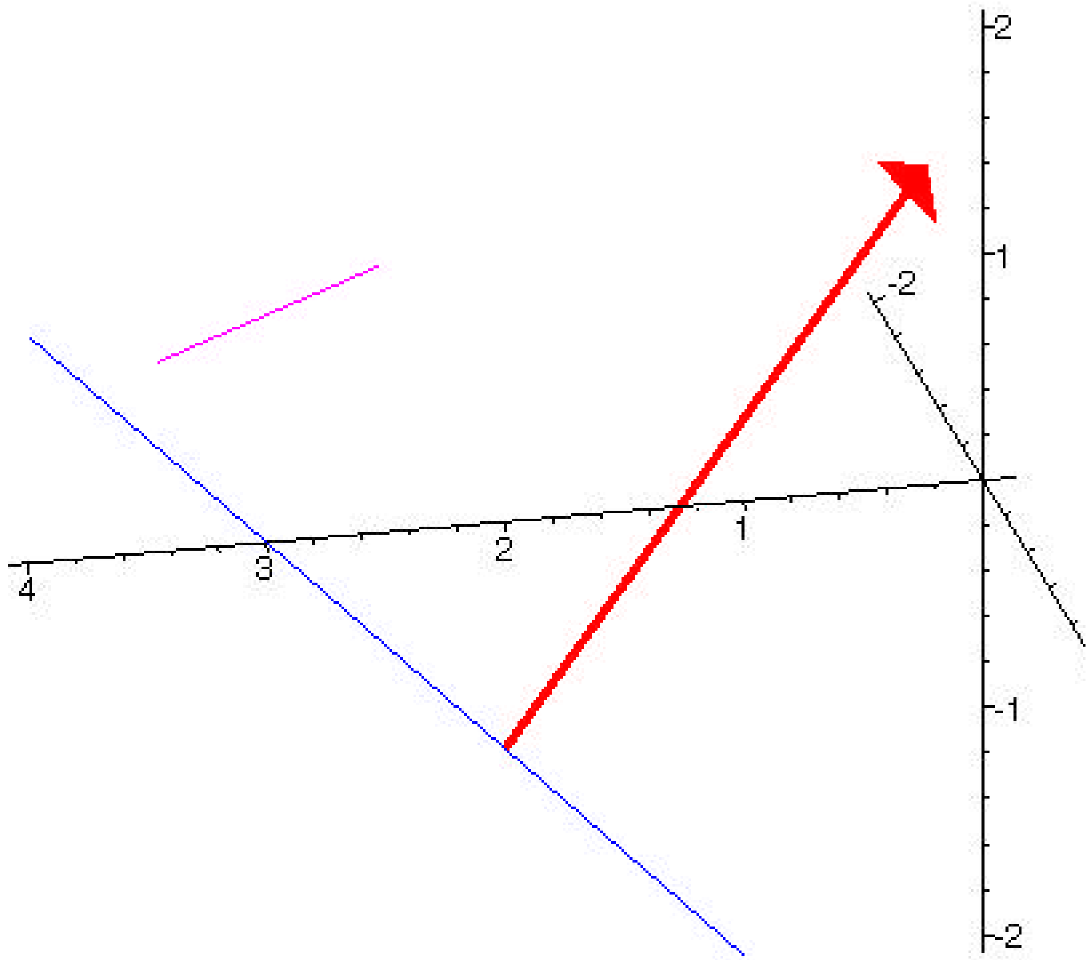
They do not intersect.

Therefore, the lines are skew.

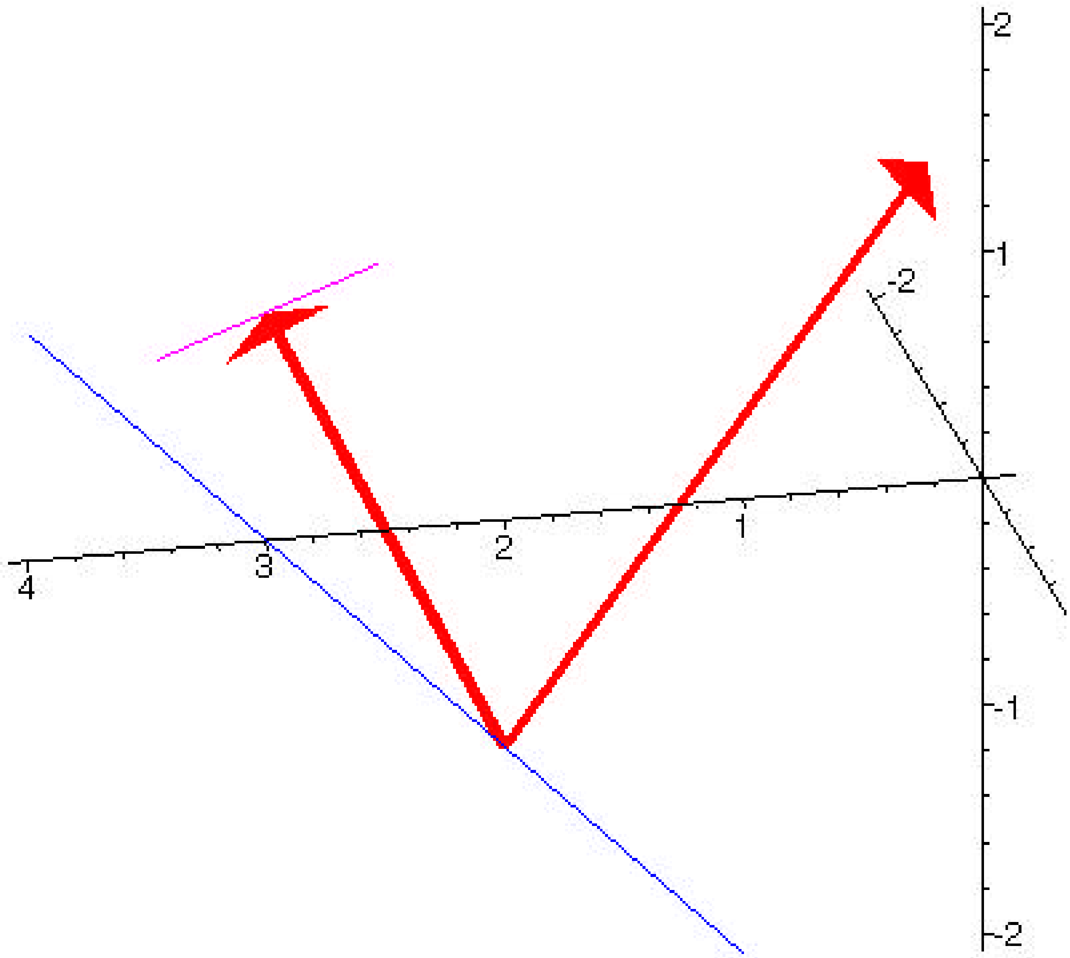
Problem 4 (1999):

(b) Calculate the distance between the two lines.

$$\ell_1 : \begin{cases} x = 2 + t \\ y = 0 \\ z = -1 + t \end{cases} \qquad \ell_2 : \begin{cases} x = 3 \\ y = 2t \\ z = 1 + t \end{cases}$$



2
1
-1
-2
-2
4
3
2
1



2
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Perpendicular to Both Lines:

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Perpendicular to Both Lines:

$$\langle 1, 0, 1 \rangle \times \langle 0, 2, 1 \rangle$$

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Perpendicular to Both Lines:

$$\langle 1, 0, 1 \rangle \times \langle 0, 2, 1 \rangle = \det \begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 1 \\ 0 & 2 & 1 \end{pmatrix}$$

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Perpendicular to Both Lines:

$$\langle 1, 0, 1 \rangle \times \langle 0, 2, 1 \rangle = \det \begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 1 \\ 0 & 2 & 1 \end{pmatrix} = \langle -2, -1, 2 \rangle$$

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Perpendicular to Both Lines: $\langle -2, -1, 2 \rangle$

Vector Between Lines:

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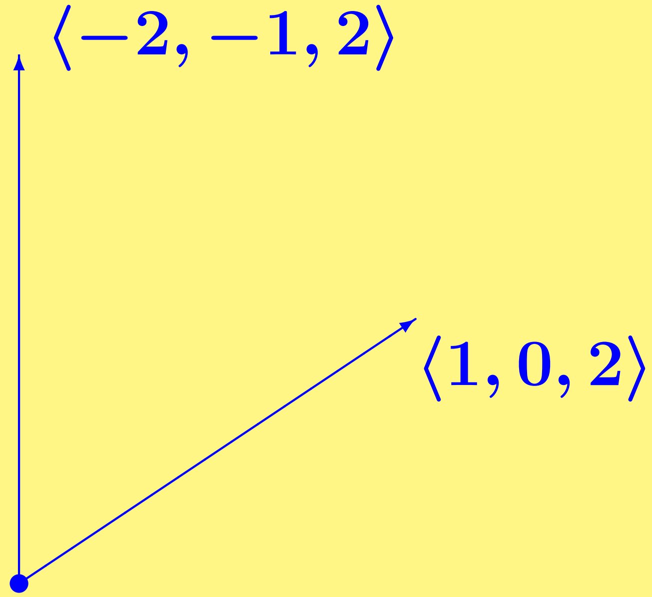
Perpendicular to Both Lines: $\langle -2, -1, 2 \rangle$

Vector Between Lines: $\langle 1, 0, 2 \rangle$

Problem 4 (1999):

(b) Calculate the distance between the two lines.



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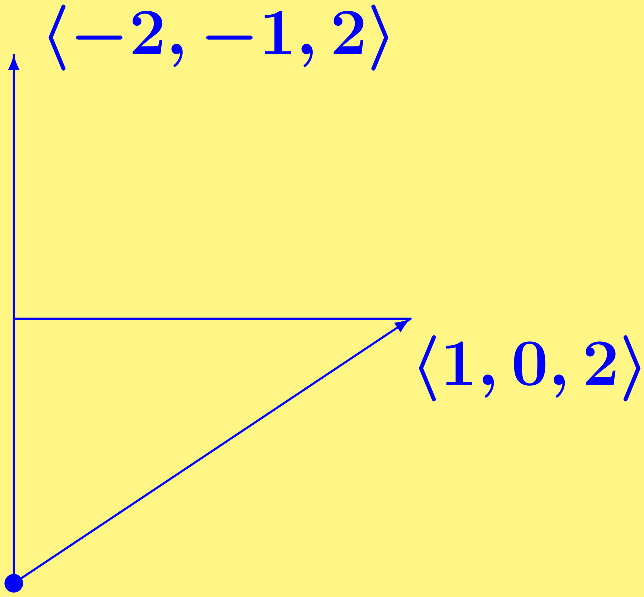

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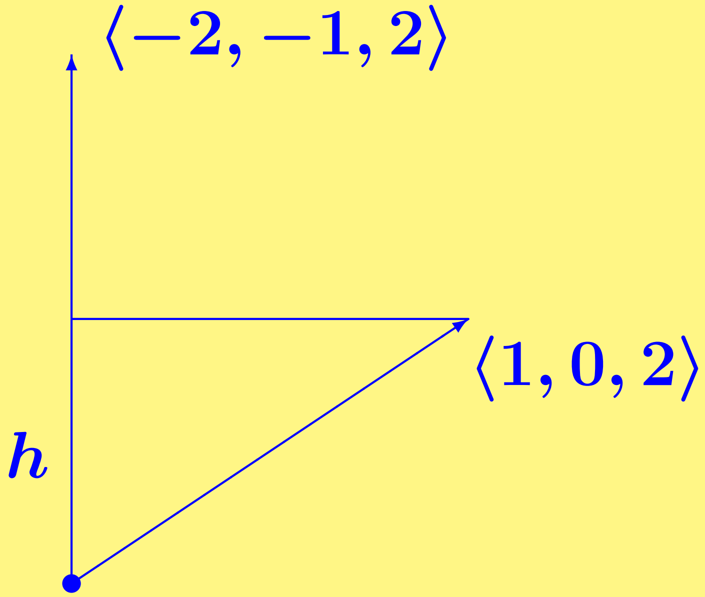

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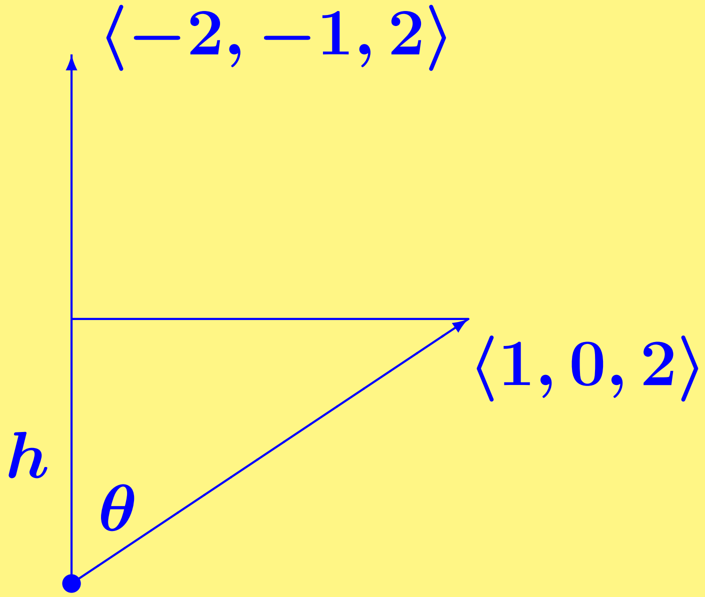
(b) Calculate the distance between the two lines.



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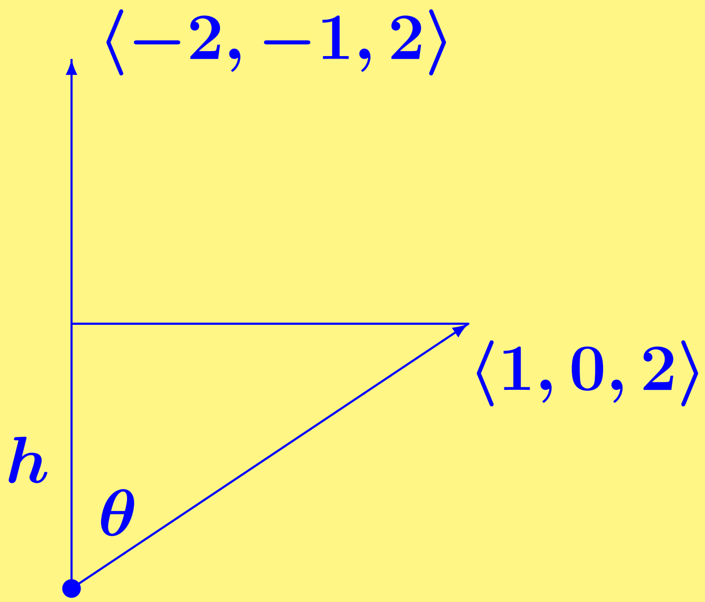
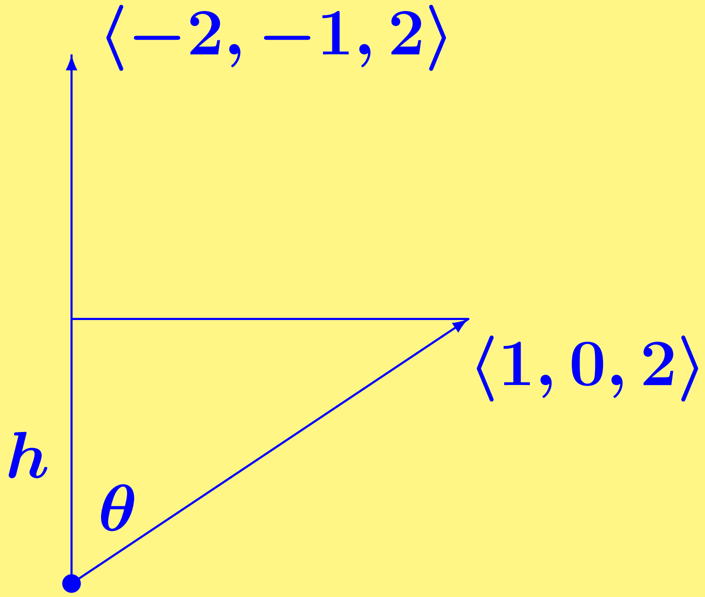


$$\frac{h}{|\langle 1, 0, 2 \rangle|} = \cos\theta$$

Problem 4 (1999):

(b) Calculate the distance between the two lines.



$$\frac{h}{|\langle 1, 0, 2\rangle|} = \cos\theta = \frac{\langle -2, -1, 2\rangle \cdot \langle 1, 0, 2\rangle}{|\langle -2, -1, 2\rangle|\,|\langle 1, 0, 2\rangle|}$$

Problem 4 (1999):

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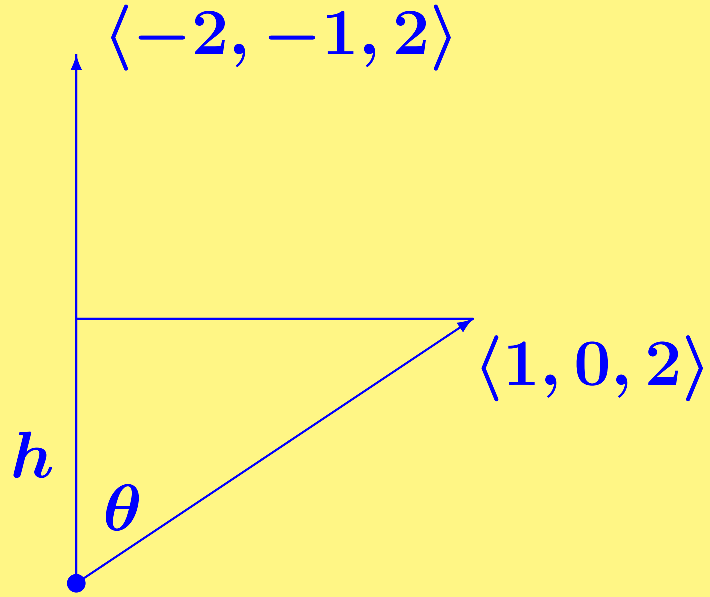

$$\frac{h}{|\langle 1, 0, 2\rangle|} = \cos\theta = \frac{2}{|\langle -2, -1, 2\rangle|\,|\langle 1, 0, 2\rangle|}$$

Problem 4 (1999):

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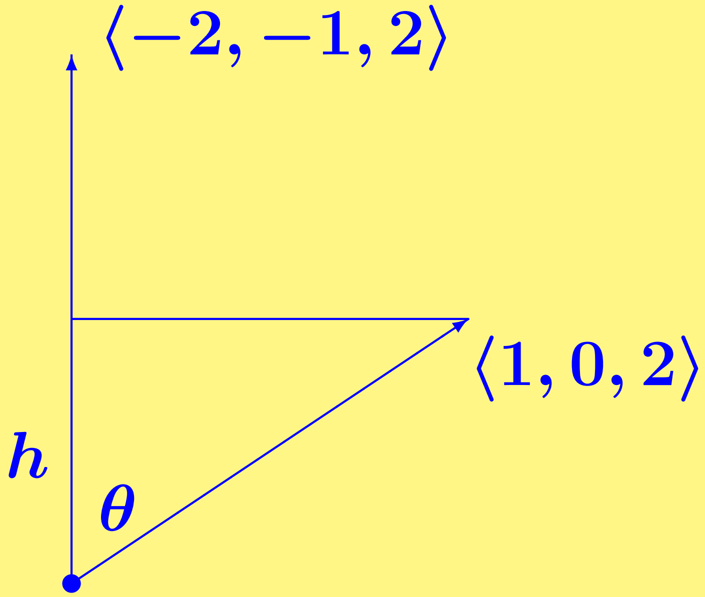

$$\frac{h}{|\langle 1, 0, 2\rangle|} = \frac{2}{\sqrt{9}\,|\langle 1, 0, 2\rangle|}$$

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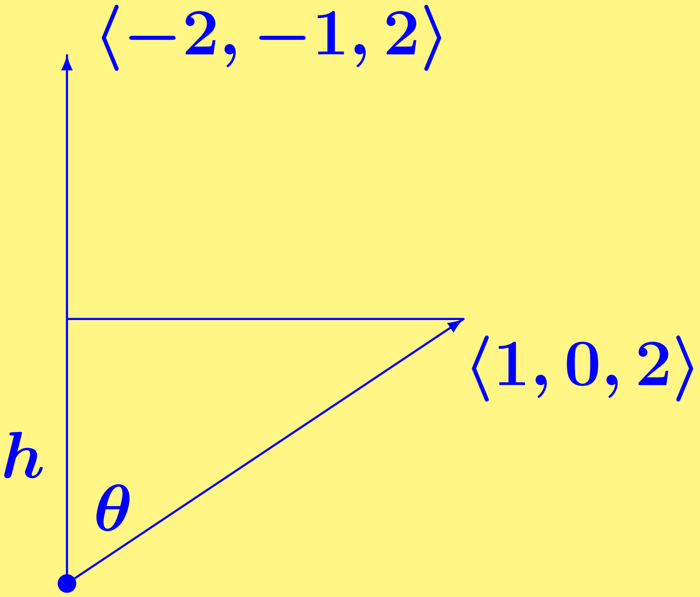

$$h = \frac{2}{3}$$

Problem 4 (1999):

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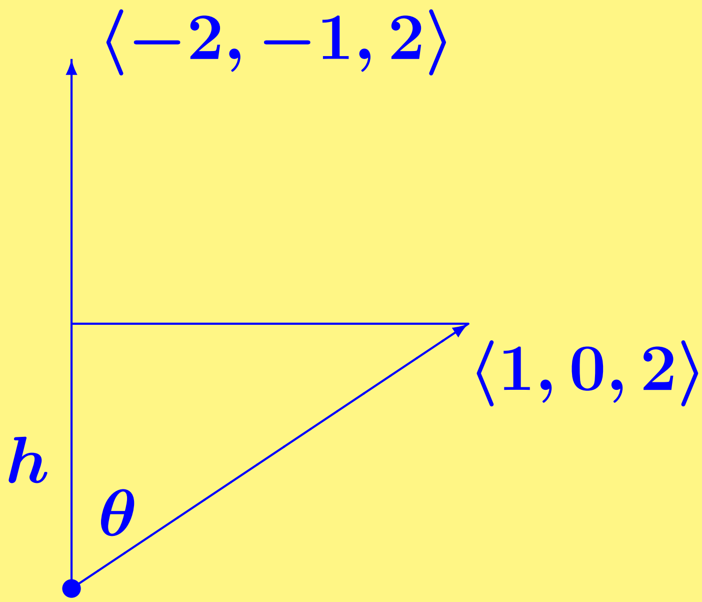

$$\boxed{h = \frac{2}{3}}$$