An Introduction to Volumes

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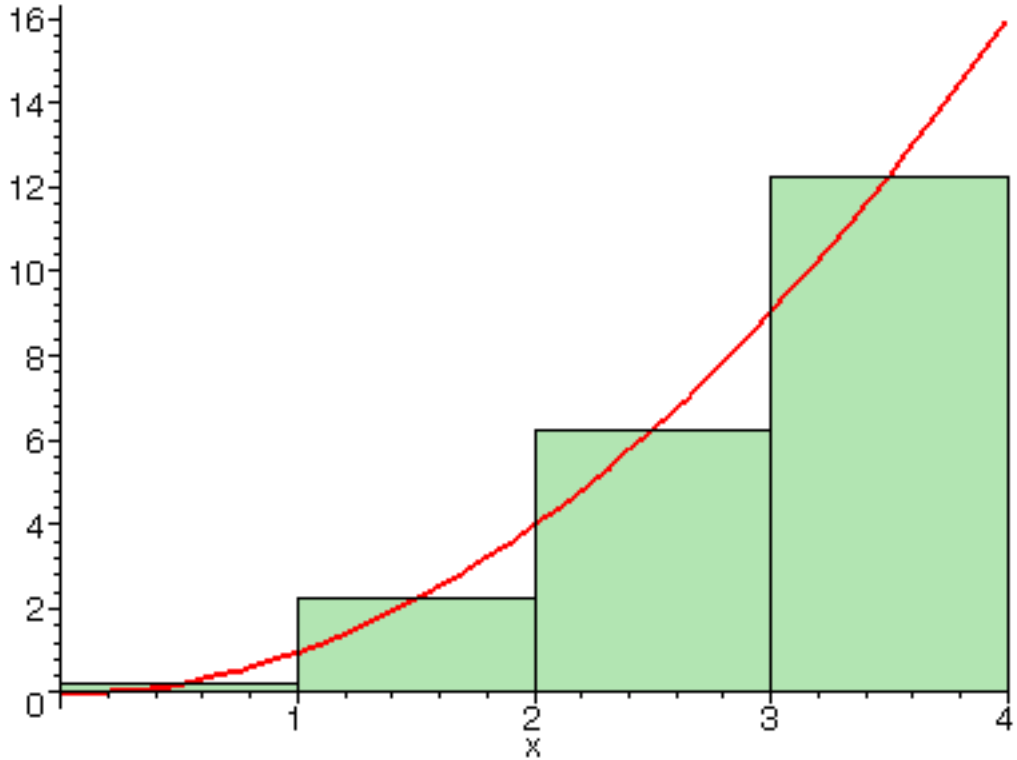
(Multiple Integrals)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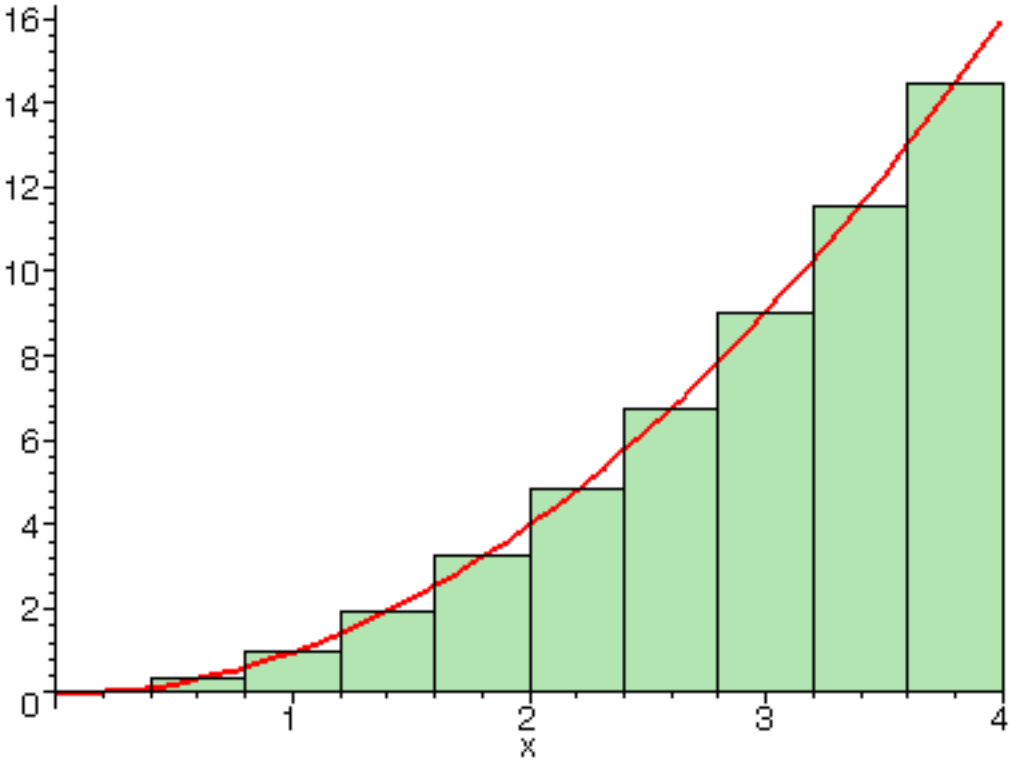
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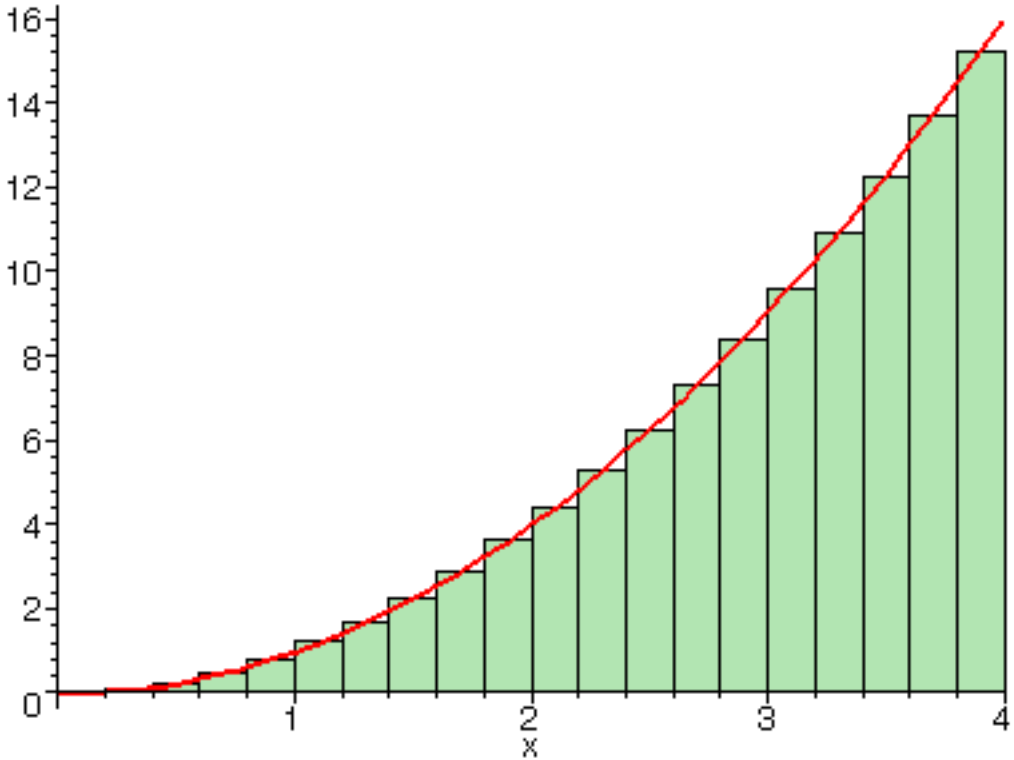
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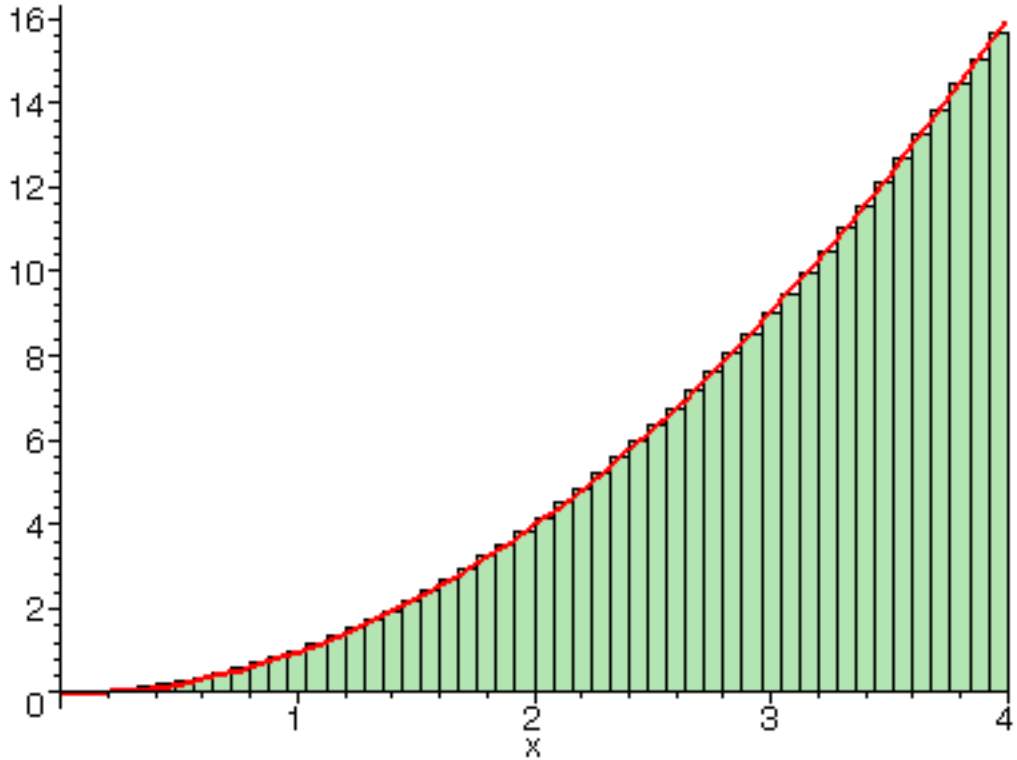
16
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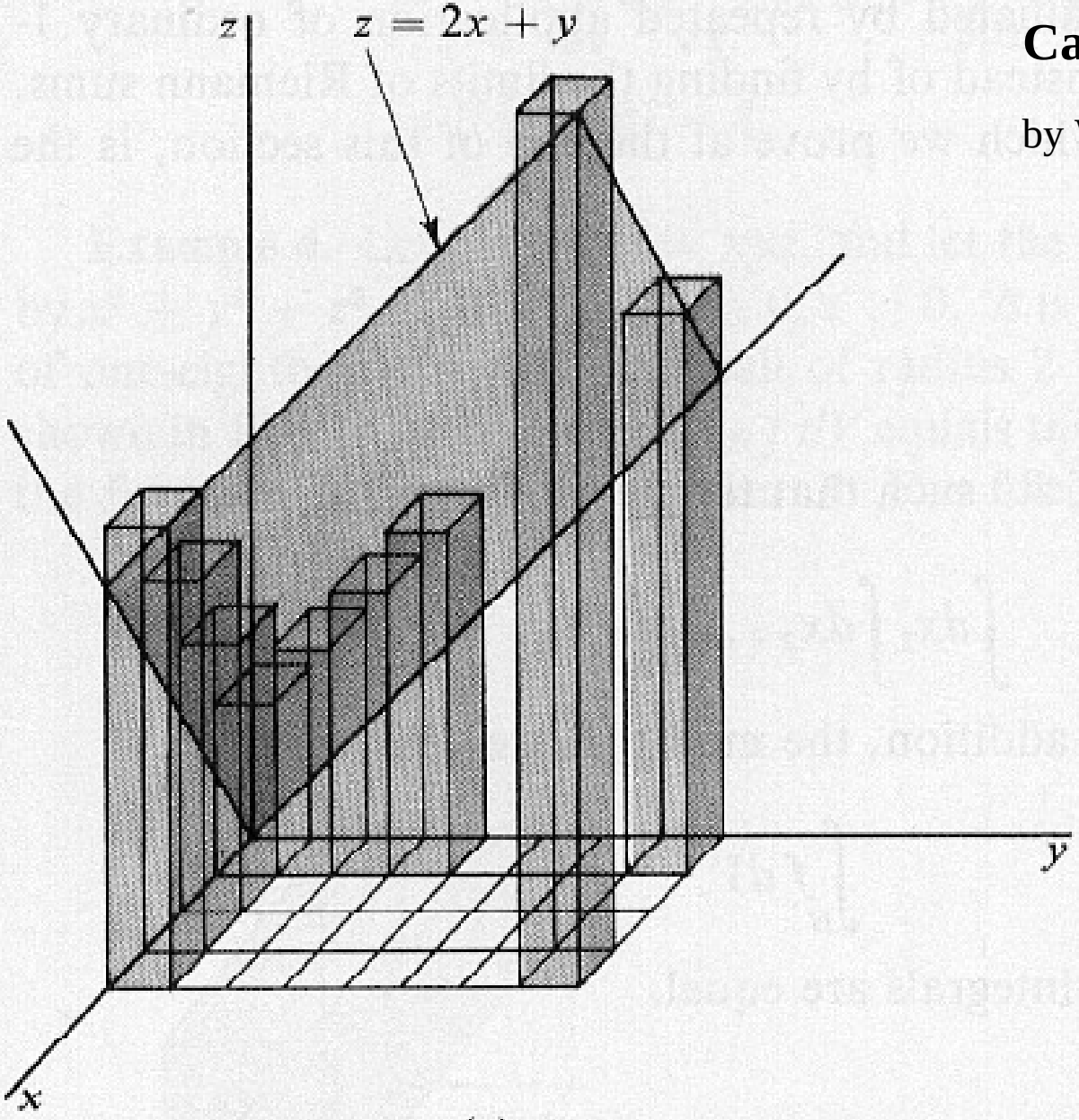
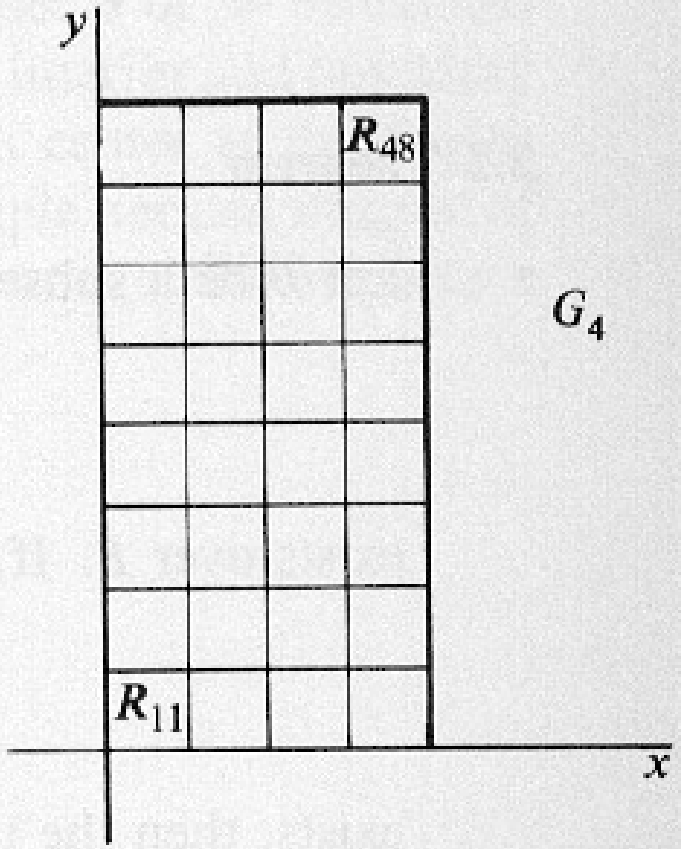
x

Calculus of Vector Functions

by Williamson, Crowell, Trotter (1972)



(a)

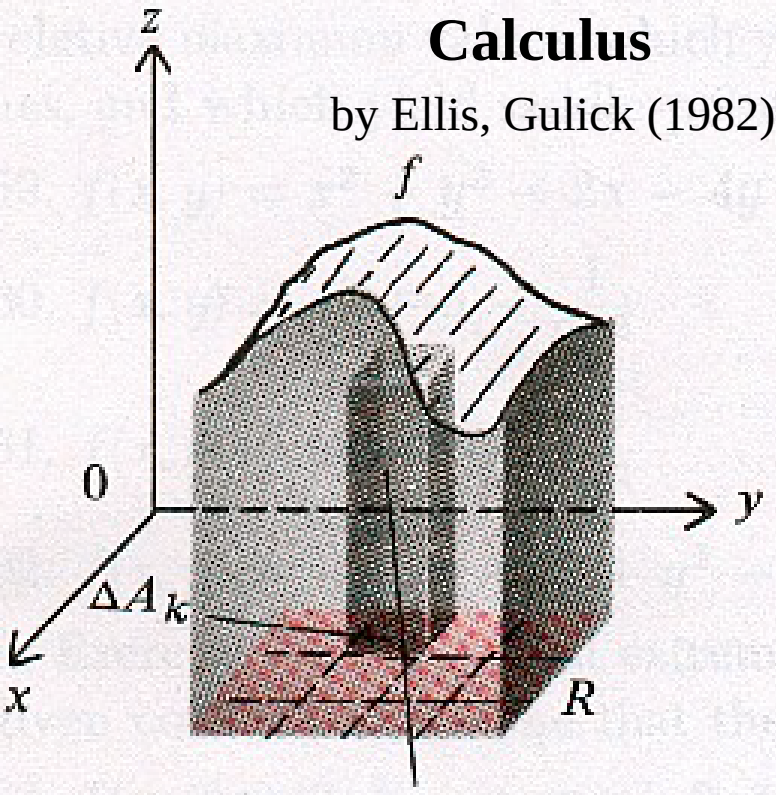


(b)

Figure 11

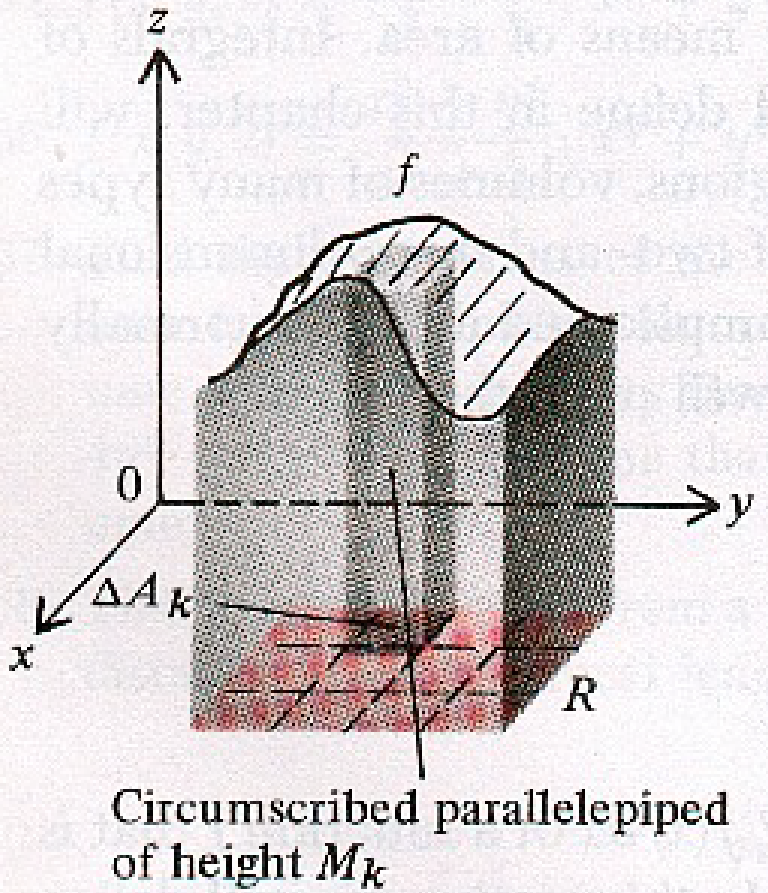
Calculus

by Ellis, Gulick (1982)



Inscribed parallelepiped of height m_k

(a)



Circumscribed parallelepiped of height M_k

(b)

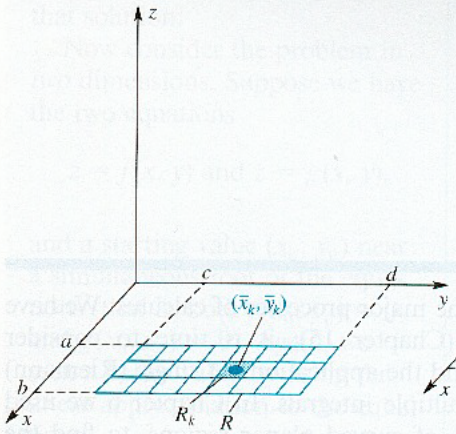


FIGURE 1

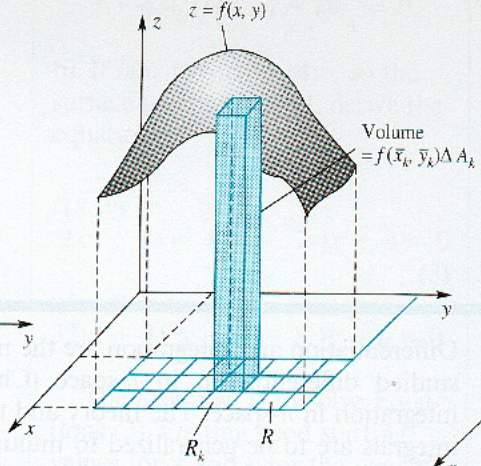


FIGURE 2

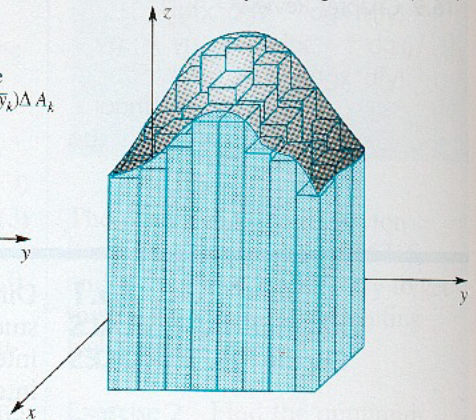


FIGURE 3

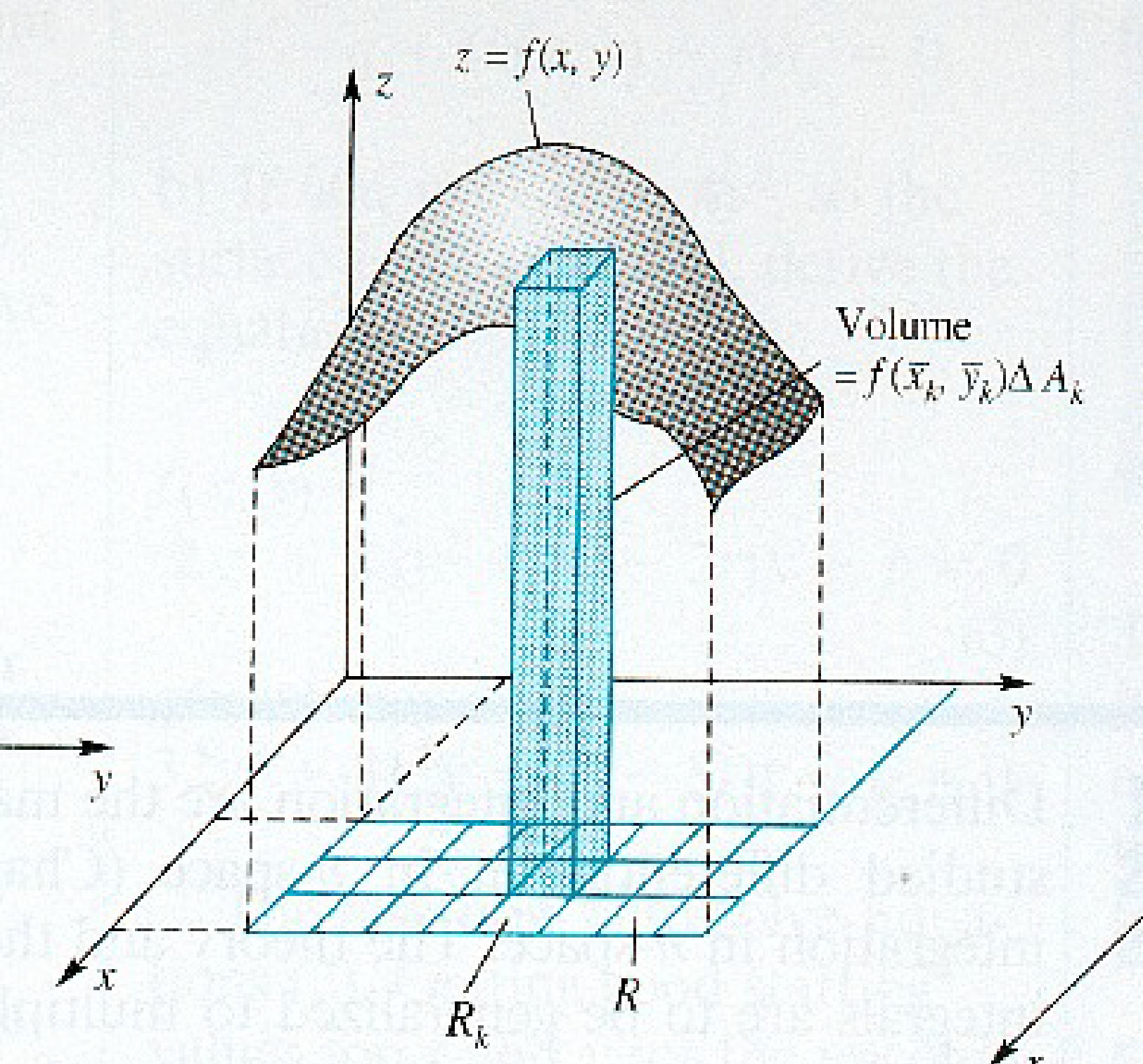


FIGURE 2

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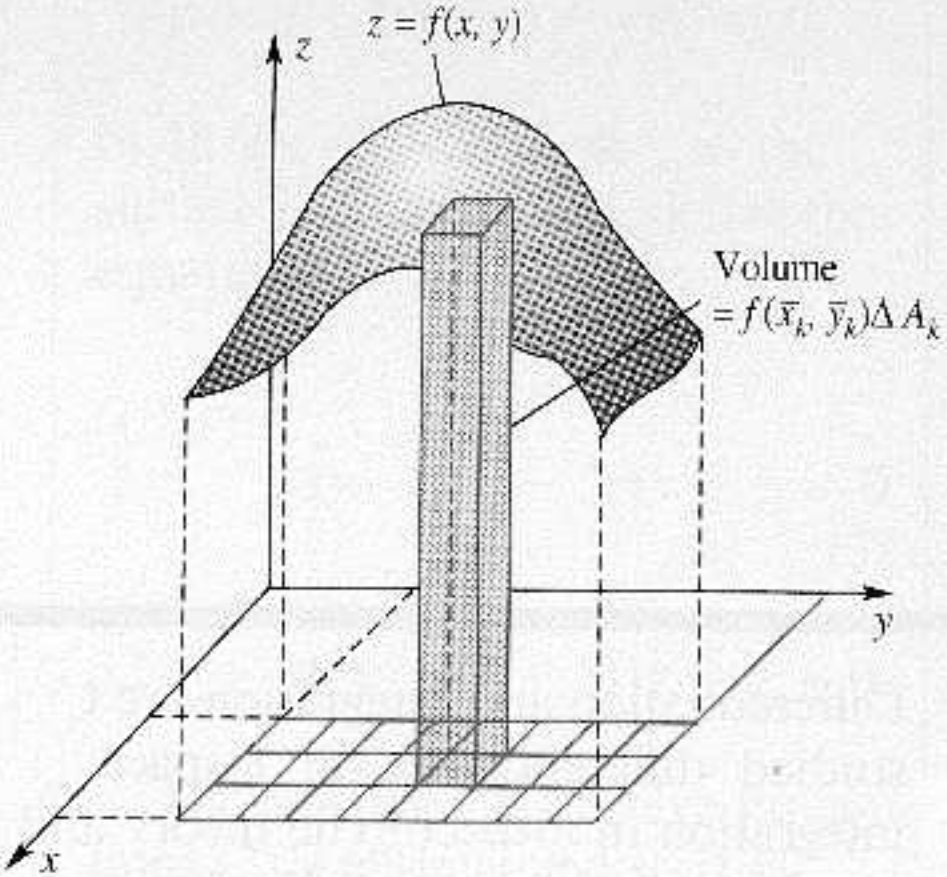

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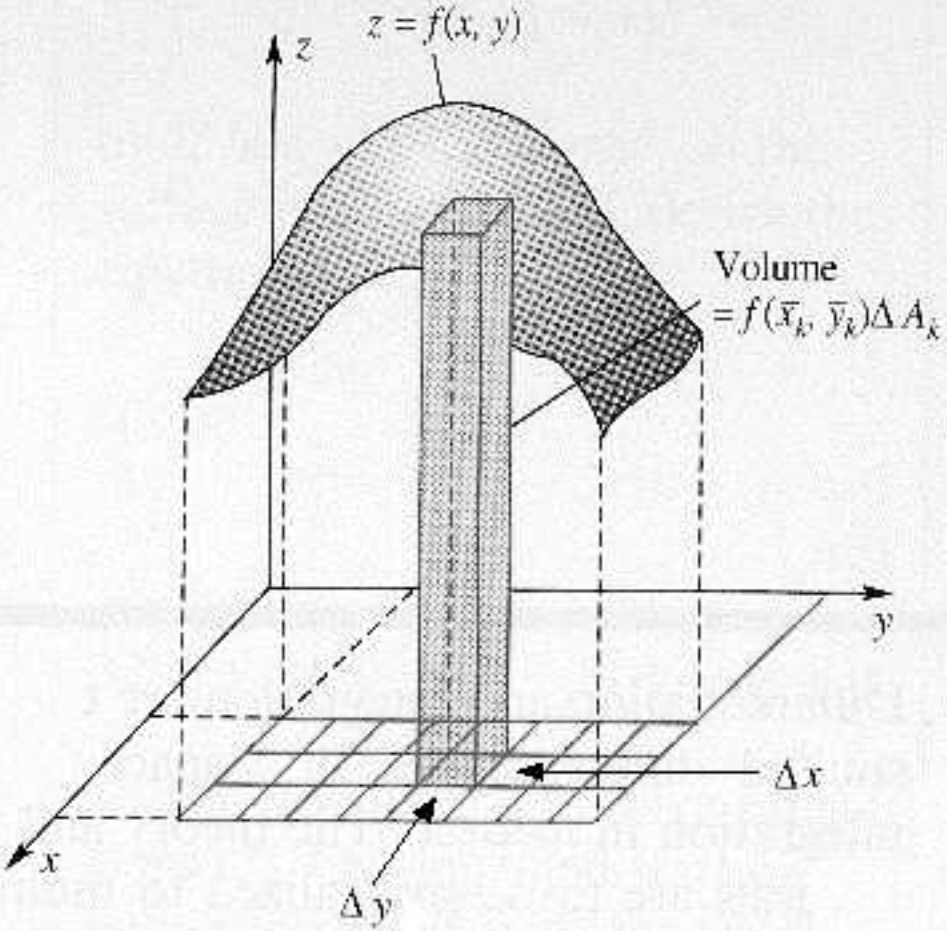

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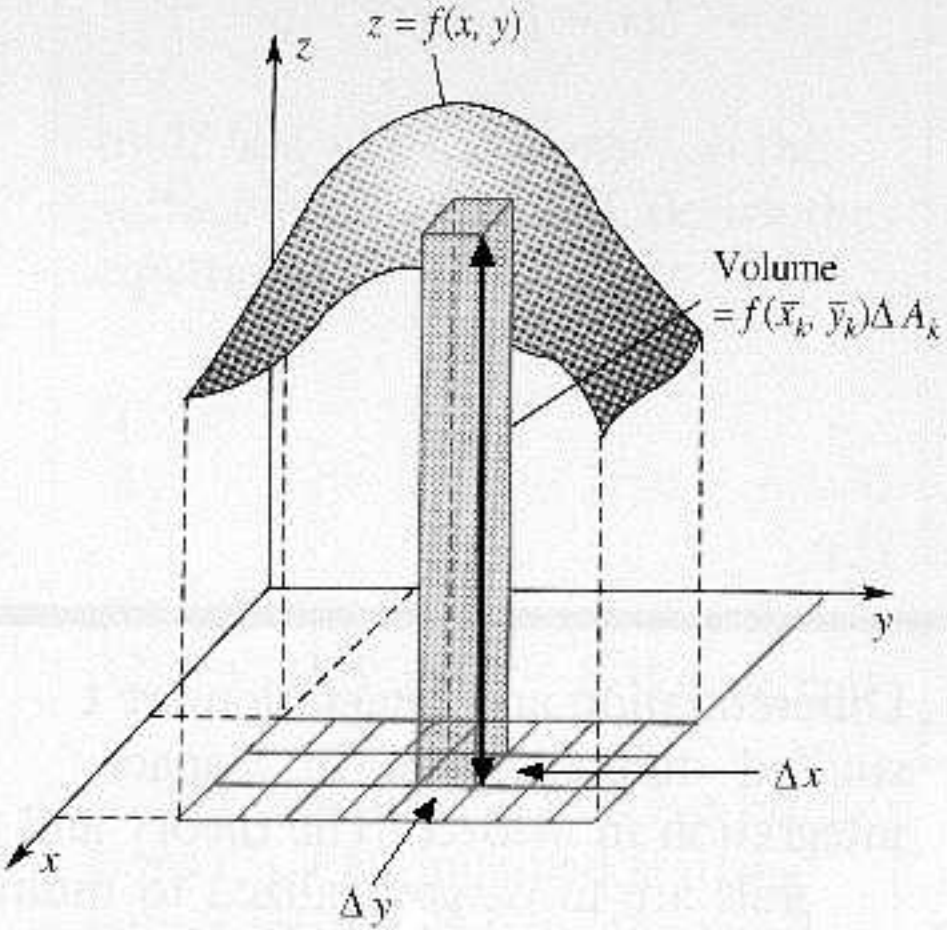

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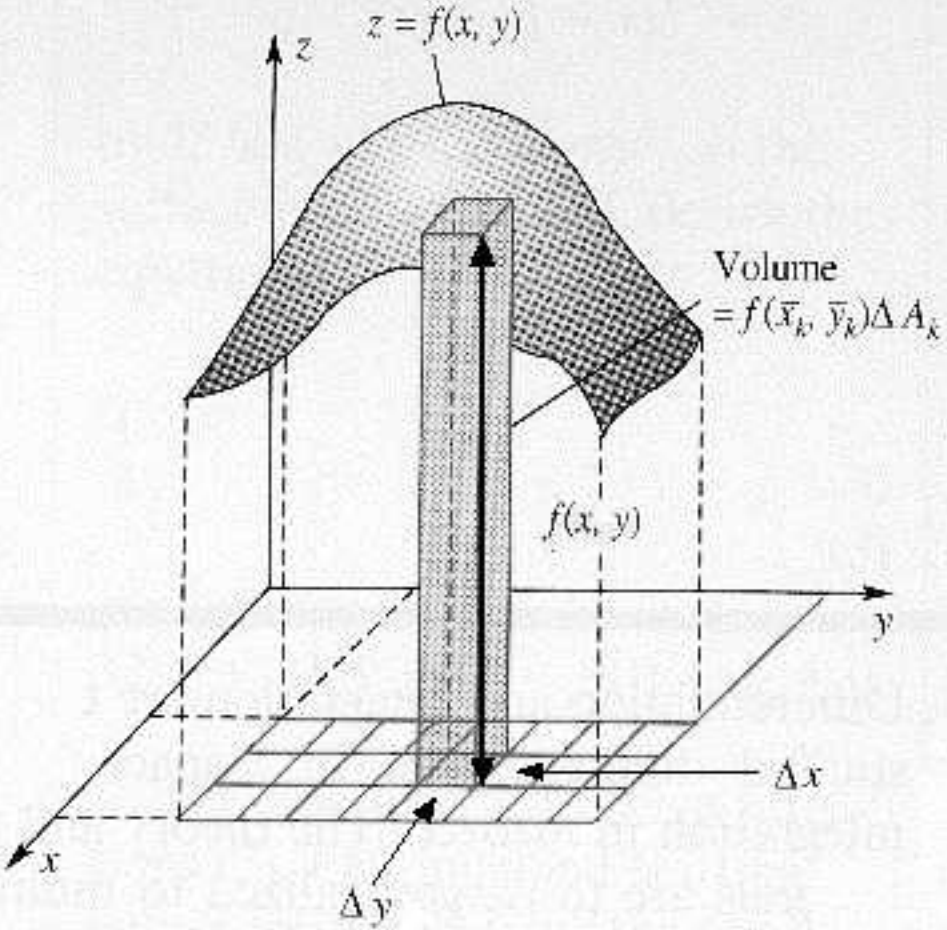

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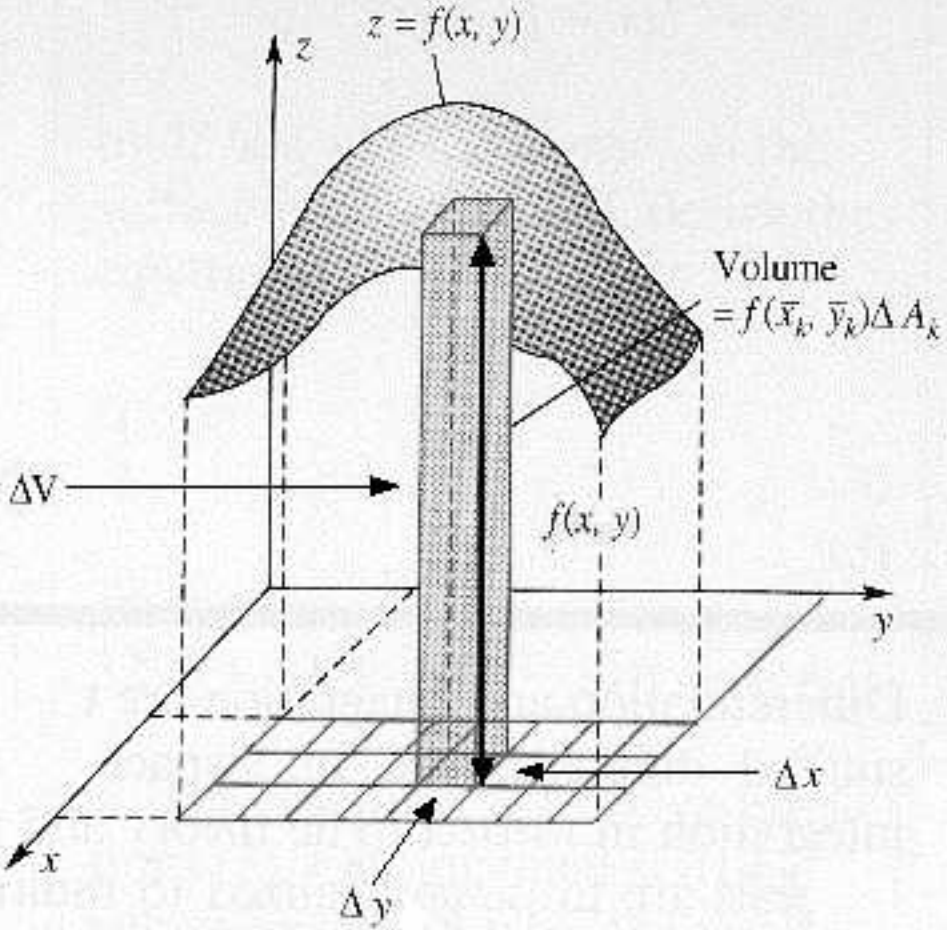

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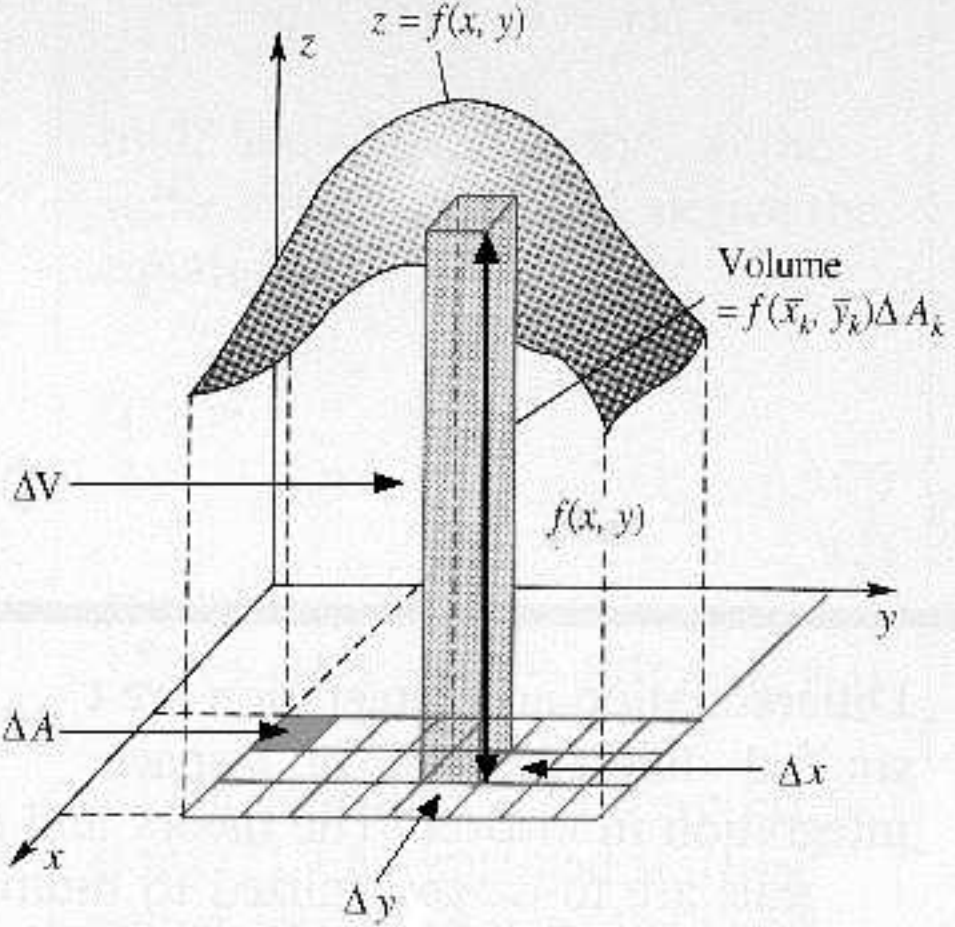

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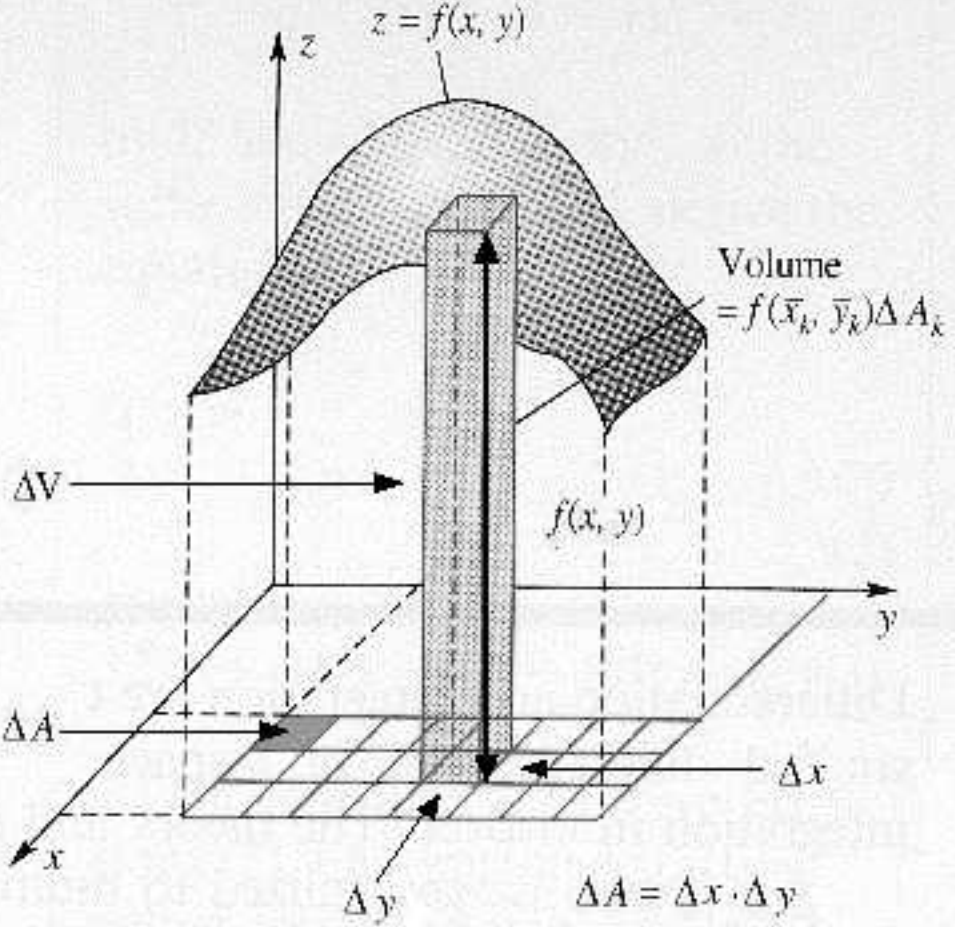

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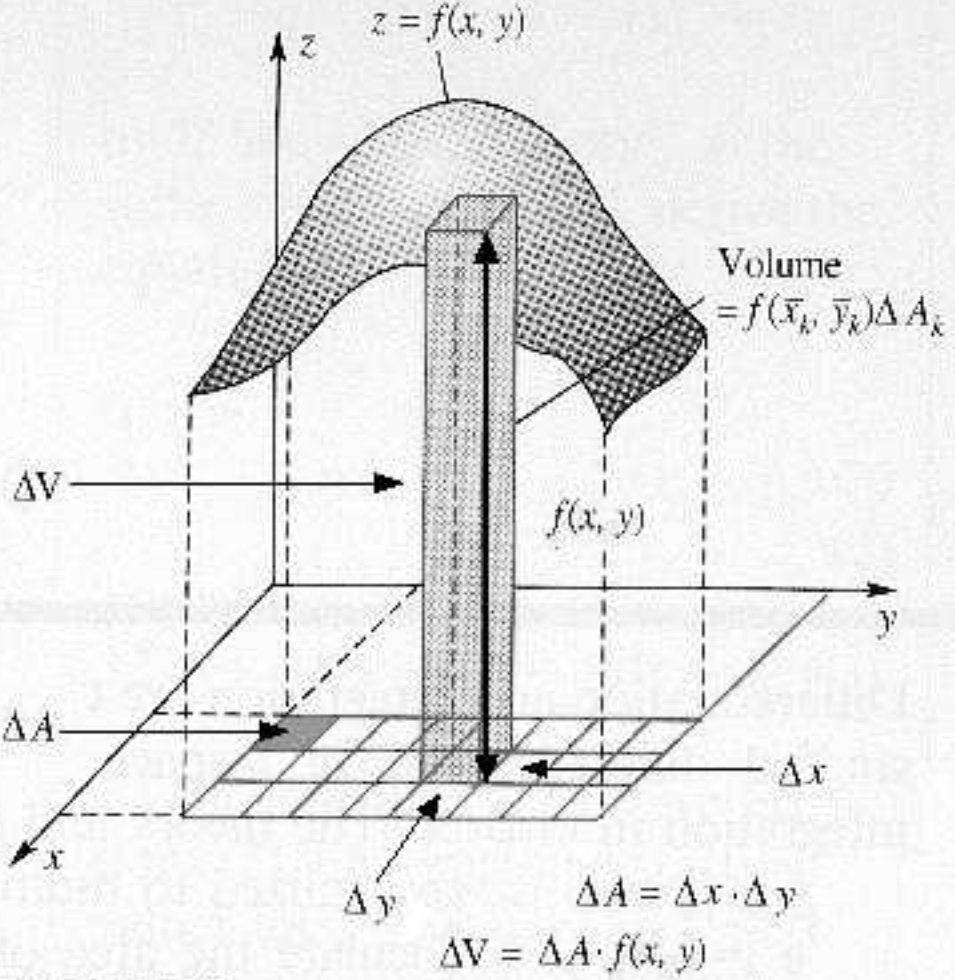

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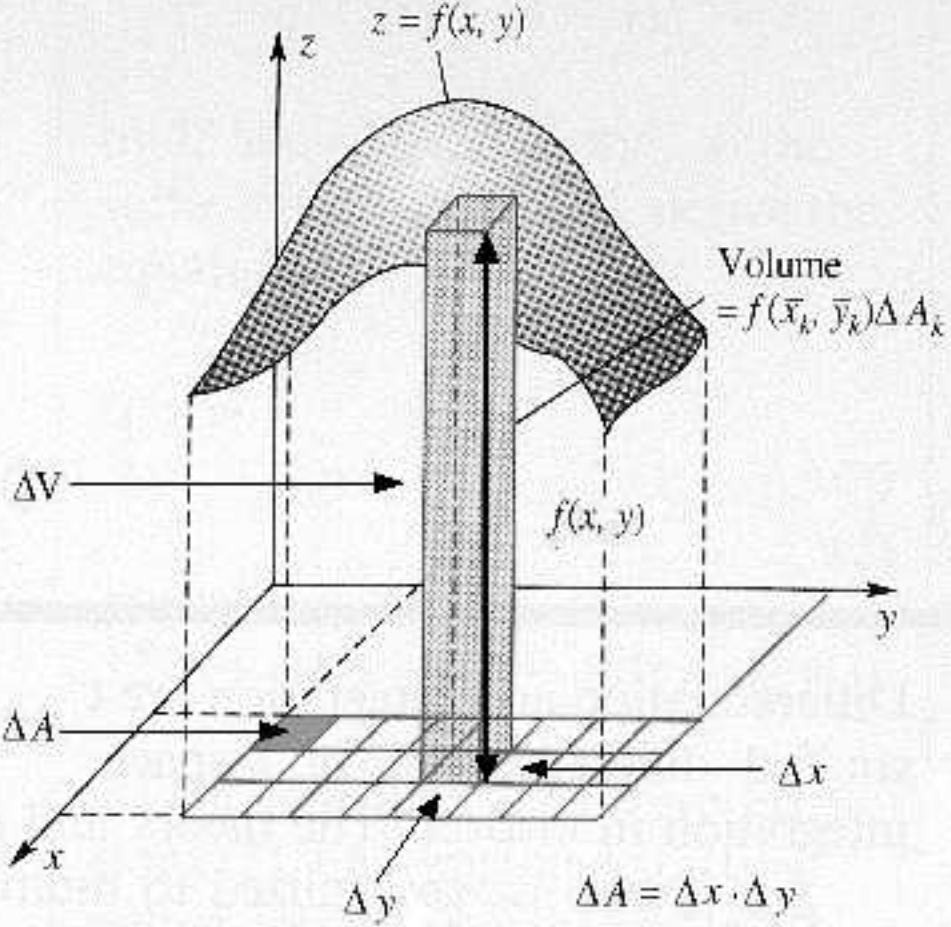
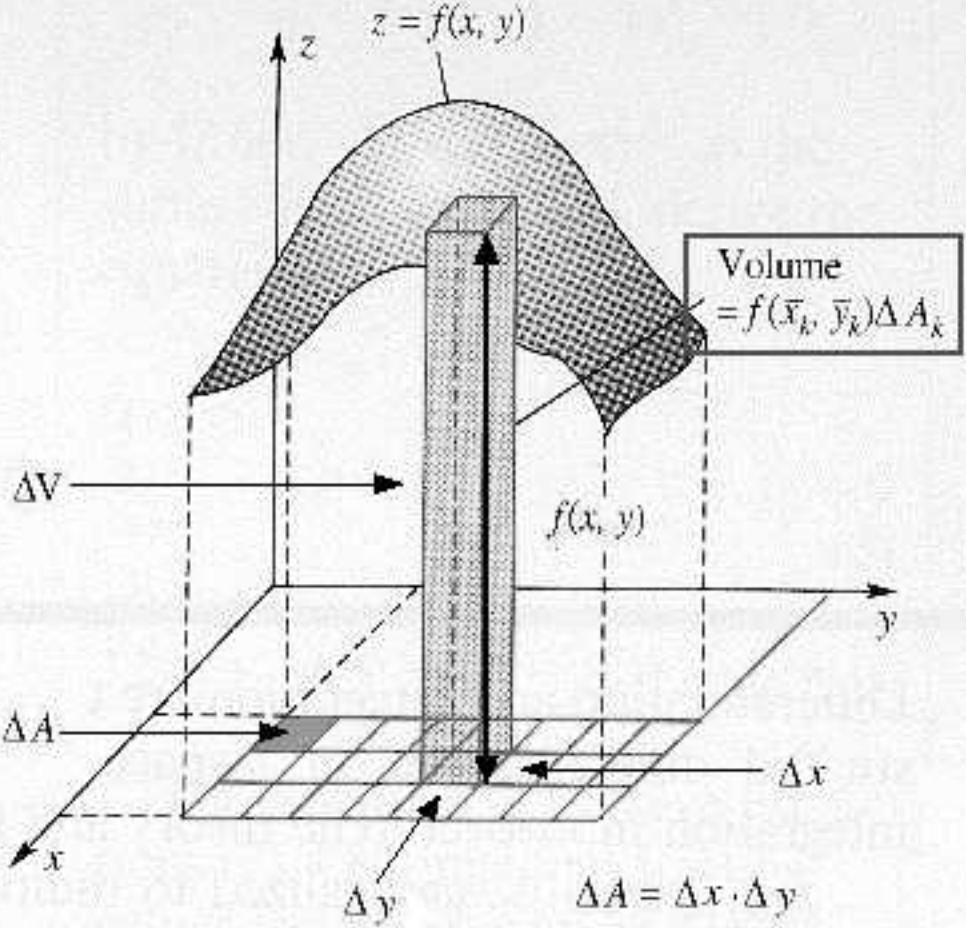

z
z = f(x, y)
Volume
= f(x̄_k, ȳ_k)ΔA_k
ΔV
f(x, y)
y
ΔA
Δx
x
Δy
ΔA = Δx · Δy
ΔV = ΔA · f(x, y)
= Δx · Δy · f(x, y)

FIGURE 2

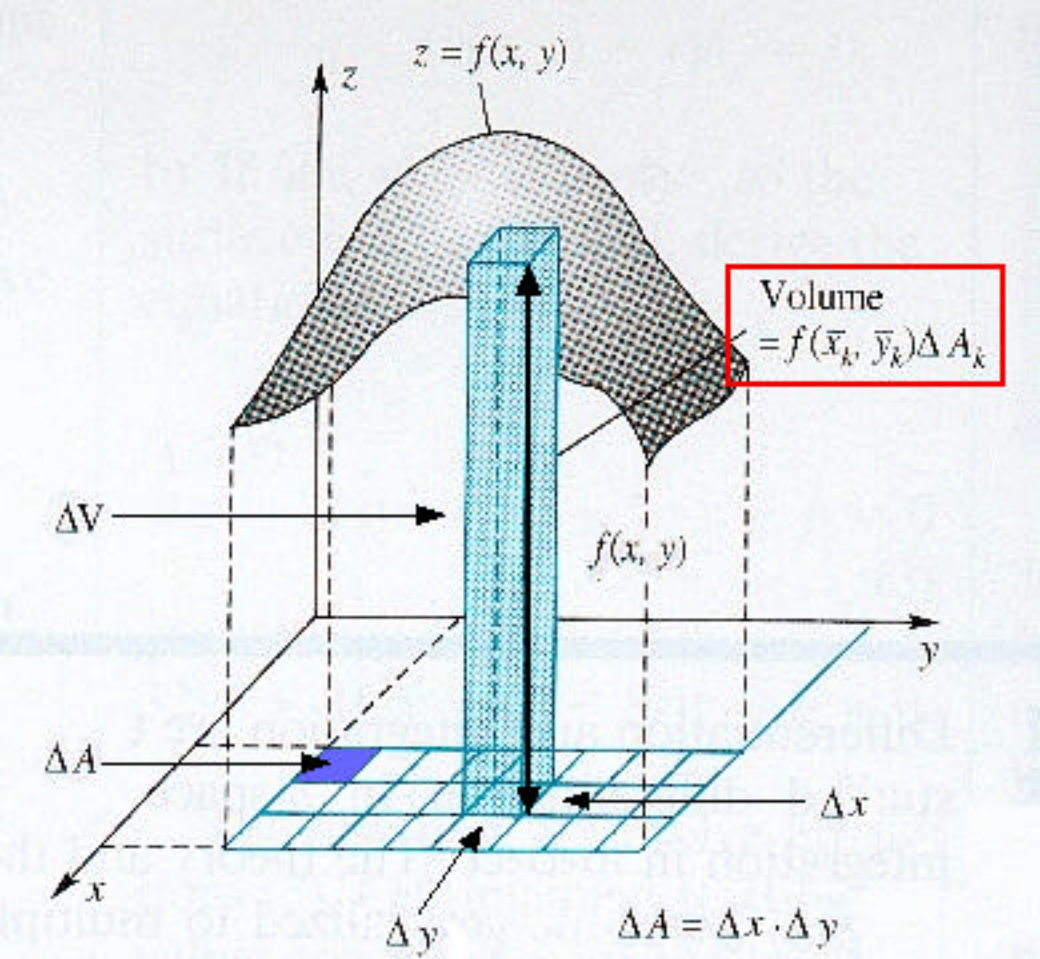


$$\Delta A = \Delta x \cdot \Delta y$$

$$\Delta V = \Delta A \cdot f(x, y)$$

$$= \Delta x \cdot \Delta y \cdot f(x, y)$$

FIGURE 2



$$\Delta A = \Delta x \cdot \Delta y$$

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$$= \Delta x \cdot \Delta y \cdot f(x, y)$$

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