

# **Max-Min Homework Problems**

## Page 759, Problem 11

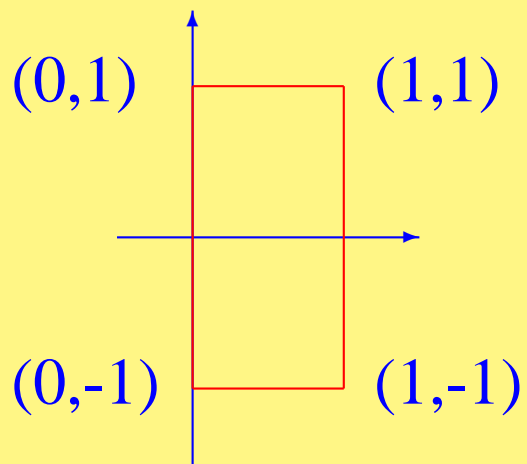
Find the maximum and minimum values of  $f$  on  $S$  where

$$f(x, y) = 3x + 4y$$

and

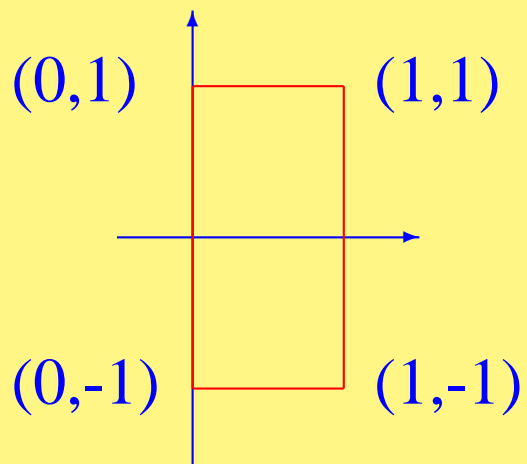
$$S = \{(x, y) : 0 \leq x \leq 1, -1 \leq y \leq 1\}$$

and indicate where they occur.



$$f(x, y) = 3x + 4y$$

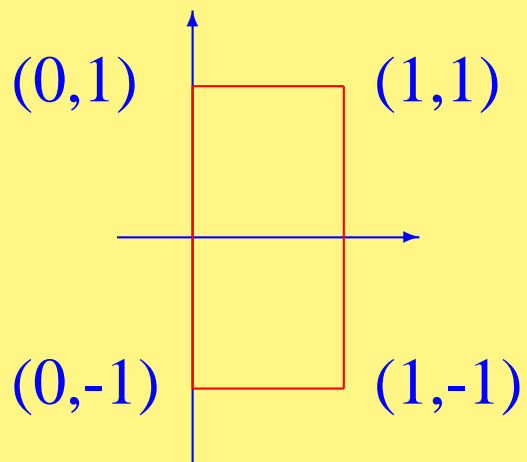
$$S = \{(x, y) : \\ 0 \leq x \leq 1, \\ -1 \leq y \leq 1\}$$



$$f(x, y) = 3x + 4y$$

$$S = \{(x, y) : \\ 0 \leq x \leq 1, \\ -1 \leq y \leq 1\}$$

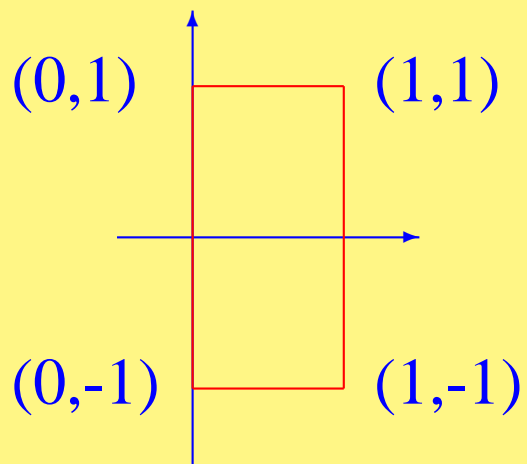
$$\nabla f =$$



$$f(x, y) = 3x + 4y$$

$$S = \{(x, y) : \\ 0 \leq x \leq 1, \\ -1 \leq y \leq 1\}$$

$$\nabla f = \langle 3, 4 \rangle$$

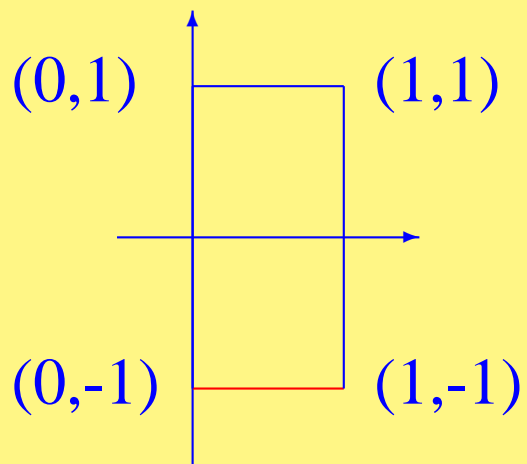


$$f(x, y) = 3x + 4y$$

$$S = \{(x, y) : \\ 0 \leq x \leq 1, \\ -1 \leq y \leq 1\}$$

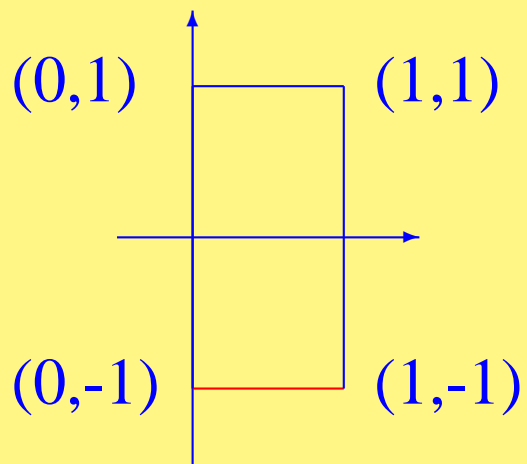
$$\nabla f = \langle 3, 4 \rangle$$

**Conclusion:** Critical points are the boundary points.



$$f(x, y) = 3x + 4y$$

$$S = \{(x, y) : \\ 0 \leq x \leq 1, \\ -1 \leq y \leq 1\}$$

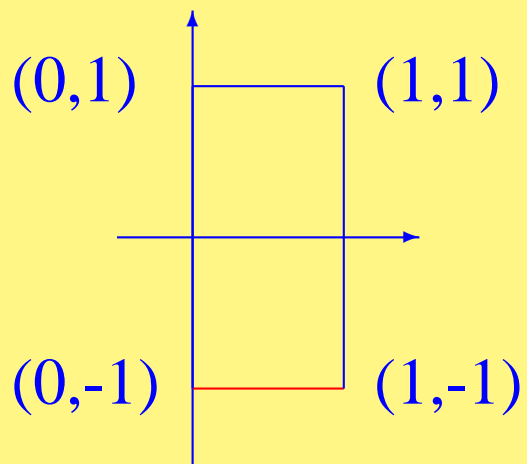


$$f(x, y) = 3x + 4y$$

$$S = \{(x, y) : \\ 0 \leq x \leq 1, \\ -1 \leq y \leq 1\}$$

$$y = -1,$$

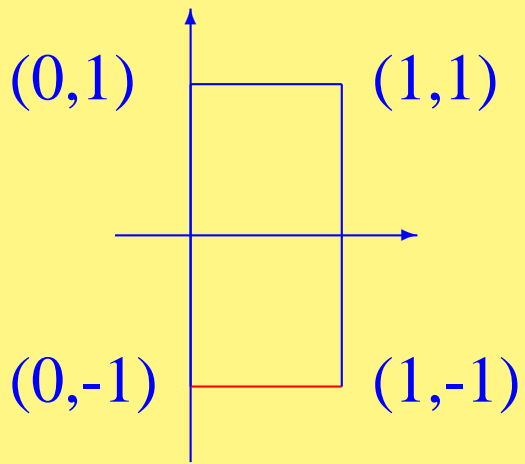




$$f(x, y) = 3x + 4y$$

$$S = \{(x, y) : \\ 0 \leq x \leq 1, \\ -1 \leq y \leq 1\}$$

$$y = -1, \quad 0 \leq x \leq 1$$

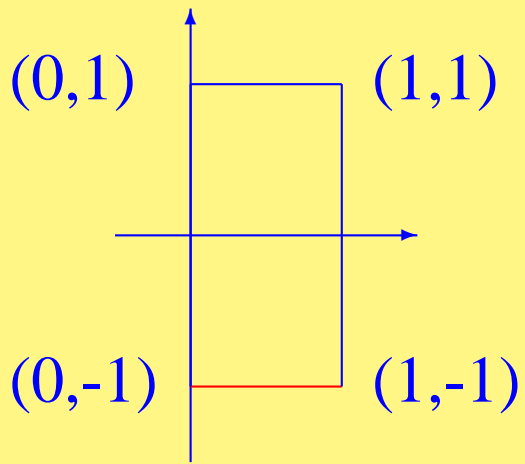


$$f(x, y) = 3x + 4y$$

$$S = \{(x, y) : \\ 0 \leq x \leq 1, \\ -1 \leq y \leq 1\}$$

$$y = -1, \quad 0 \leq x \leq 1$$

$$g(x) = 3x - 4,$$

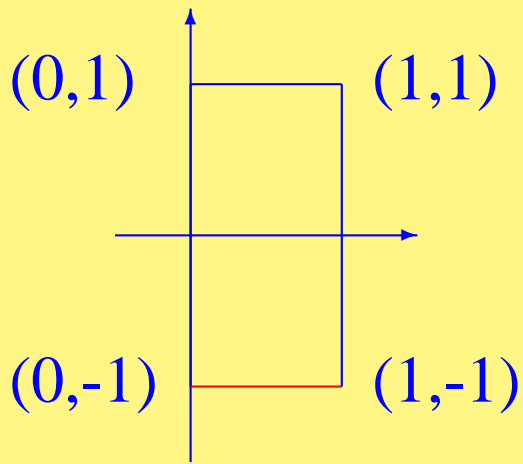


$$f(x, y) = 3x + 4y$$

$$S = \{(x, y) : \\ 0 \leq x \leq 1, \\ -1 \leq y \leq 1\}$$

$$y = -1, \quad 0 \leq x \leq 1$$

$$g(x) = 3x - 4, \quad g'(x) = 3$$



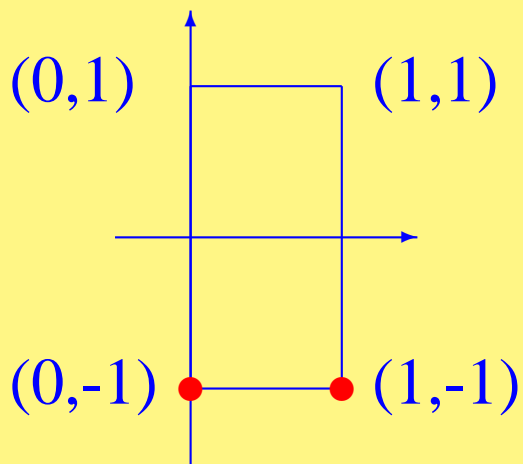
$$f(x, y) = 3x + 4y$$

$$S = \{(x, y) : \\ 0 \leq x \leq 1, \\ -1 \leq y \leq 1\}$$

$$y = -1, \quad 0 \leq x \leq 1$$

$$g(x) = 3x - 4, \quad g'(x) = 3$$

**Conclusion:** Critical points are at  $x = 0$  and  $x = 1$ .



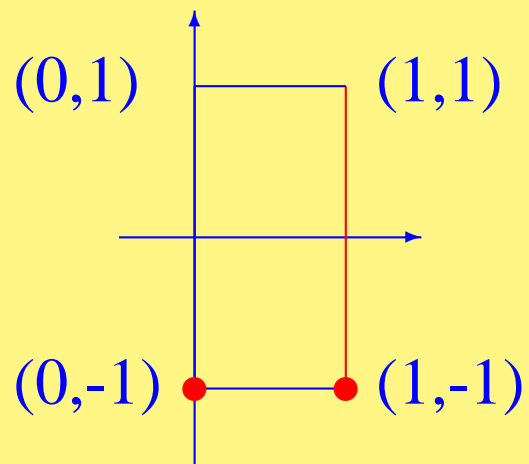
$$f(x, y) = 3x + 4y$$

$$S = \{(x, y) : \\ 0 \leq x \leq 1, \\ -1 \leq y \leq 1\}$$

$$y = -1, \quad 0 \leq x \leq 1$$

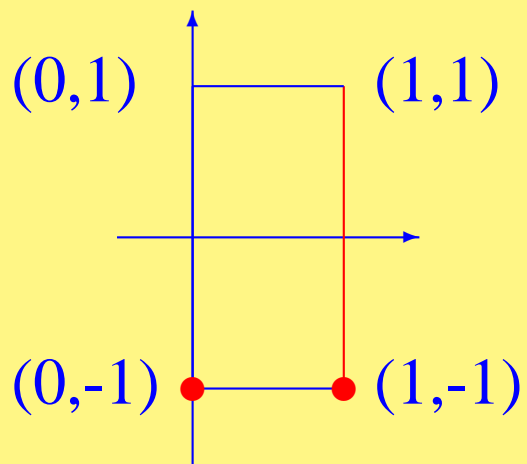
$$g(x) = 3x - 4, \quad g'(x) = 3$$

**Conclusion:** Critical points are at  $x = 0$  and  $x = 1$ .



$$f(x, y) = 3x + 4y$$

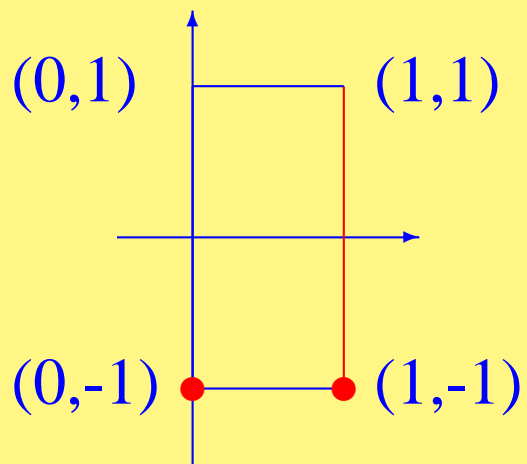
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$$S = \{(x, y) : \\ 0 \leq x \leq 1, \\ -1 \leq y \leq 1\}$$

$$x = 1,$$

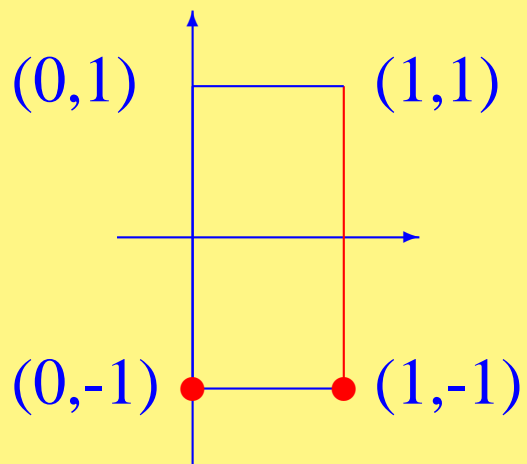


$$f(x, y) = 3x + 4y$$

$$S = \{(x, y) : \\ 0 \leq x \leq 1, \\ -1 \leq y \leq 1\}$$

$$x = 1, \quad -1 \leq y \leq 1$$



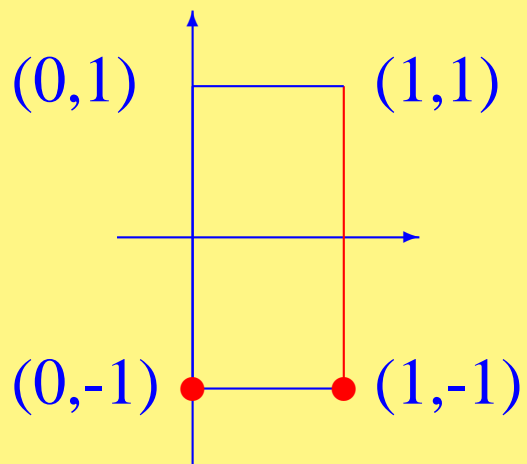


$$f(x, y) = 3x + 4y$$

$$S = \{(x, y) : \\ 0 \leq x \leq 1, \\ -1 \leq y \leq 1\}$$

$$x = 1, \quad -1 \leq y \leq 1$$

$$g(y) = 3 + 4y,$$

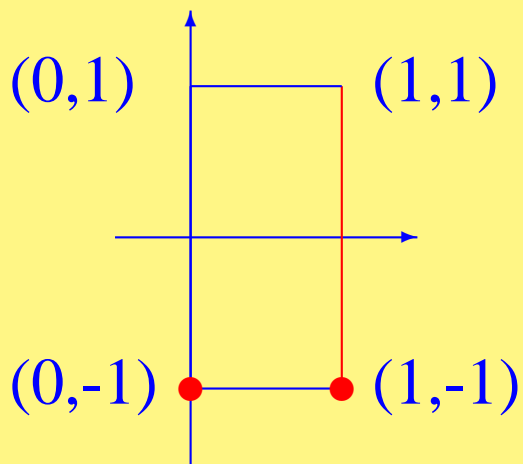


$$f(x, y) = 3x + 4y$$

$$S = \{(x, y) : \\ 0 \leq x \leq 1, \\ -1 \leq y \leq 1\}$$

$$x = 1, \quad -1 \leq y \leq 1$$

$$g(y) = 3 + 4y, \quad g'(y) = 4$$



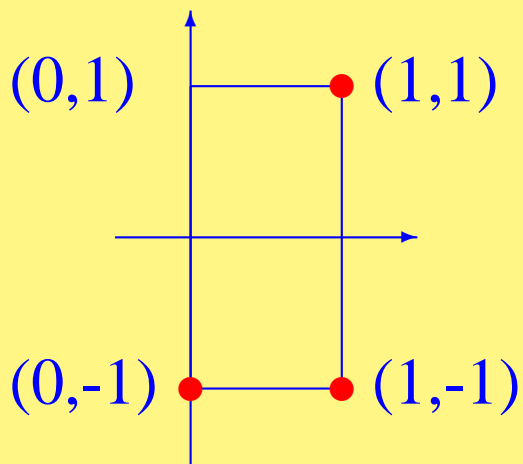
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**Conclusion:** Critical points are at  $y = -1$  and  $y = 1$ .



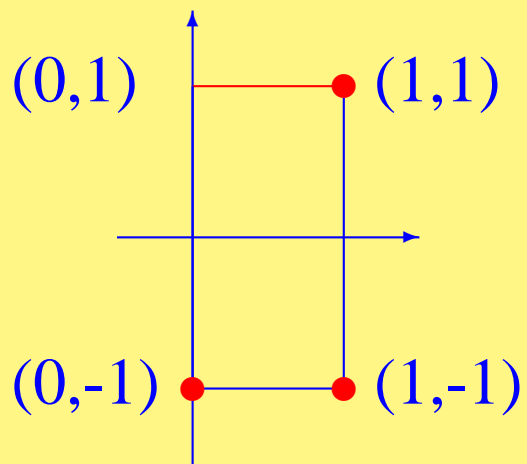
$$f(x, y) = 3x + 4y$$

$$S = \{(x, y) : \\ 0 \leq x \leq 1, \\ -1 \leq y \leq 1\}$$

$$x = 1, \quad -1 \leq y \leq 1$$

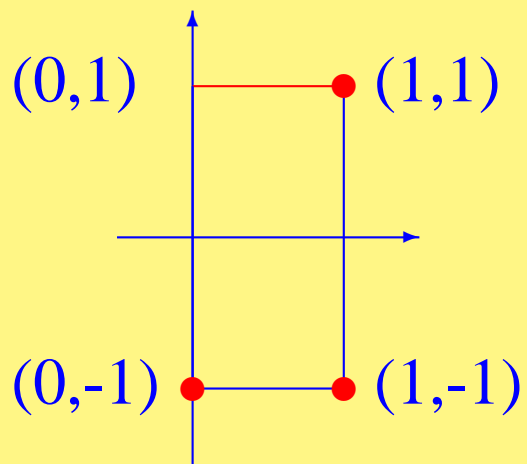
$$g(y) = 3 + 4y, \quad g'(y) = 4$$

**Conclusion:** Critical points are at  $y = -1$  and  $y = 1$ .



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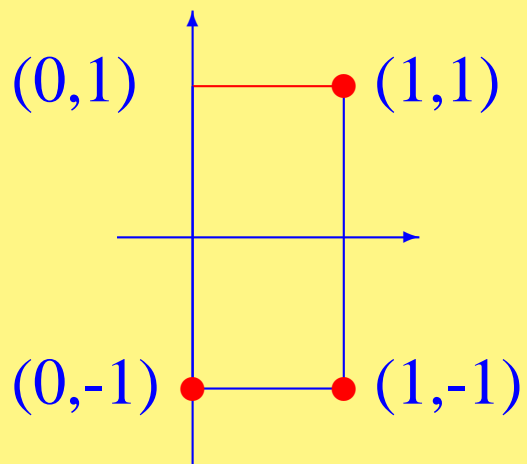
$$S = \{(x, y) : \\ 0 \leq x \leq 1, \\ -1 \leq y \leq 1\}$$



$$f(x, y) = 3x + 4y$$

$$S = \{(x, y) : \\ 0 \leq x \leq 1, \\ -1 \leq y \leq 1\}$$

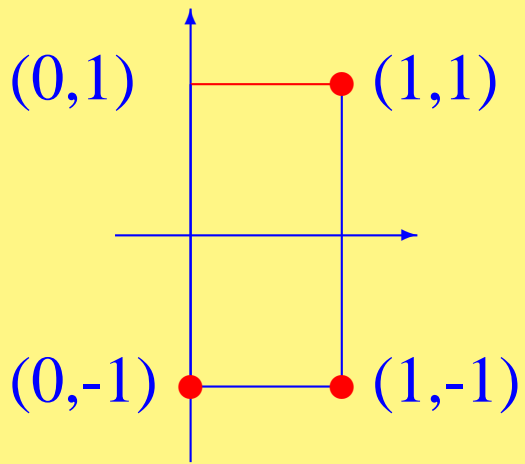
$$y = 1,$$



$$f(x, y) = 3x + 4y$$

$$S = \{(x, y) : \\ 0 \leq x \leq 1, \\ -1 \leq y \leq 1\}$$

$$y = 1, \quad 0 \leq x \leq 1$$



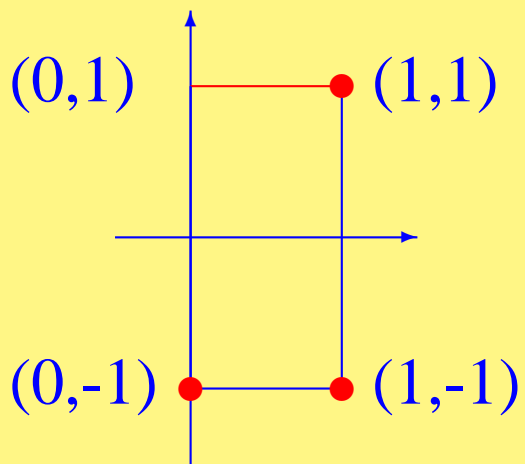
$$f(x, y) = 3x + 4y$$

$$S = \{(x, y) : \\ 0 \leq x \leq 1, \\ -1 \leq y \leq 1\}$$

$$y = 1, \quad 0 \leq x \leq 1$$

$$g(x) = 3x + 4,$$



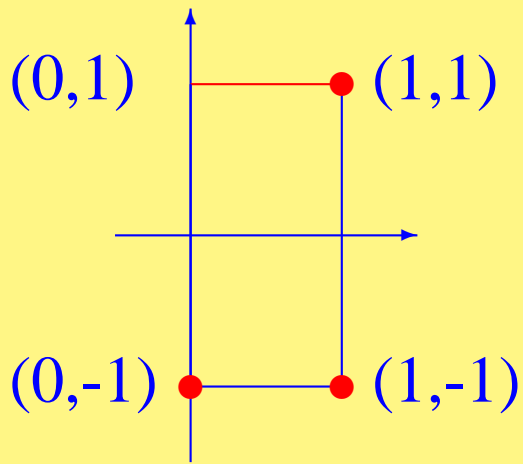


$$f(x, y) = 3x + 4y$$

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$$y = 1, \quad 0 \leq x \leq 1$$

$$g(x) = 3x + 4, \quad g'(x) = 3$$



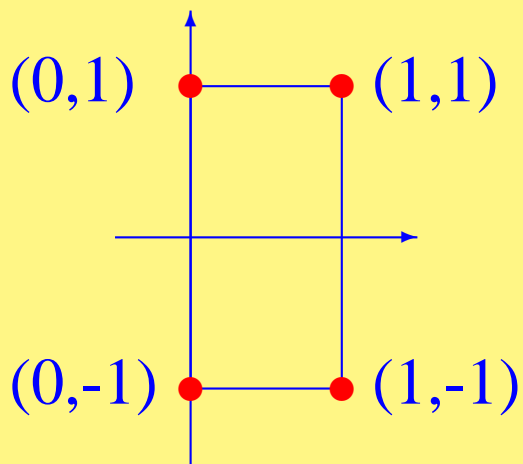
$$f(x, y) = 3x + 4y$$

$$S = \{(x, y) : \\ 0 \leq x \leq 1, \\ -1 \leq y \leq 1\}$$

$$y = 1, \quad 0 \leq x \leq 1$$

$$g(x) = 3x + 4, \quad g'(x) = 3$$

**Conclusion:** Critical points are at  $x = 0$  and  $x = 1$ .



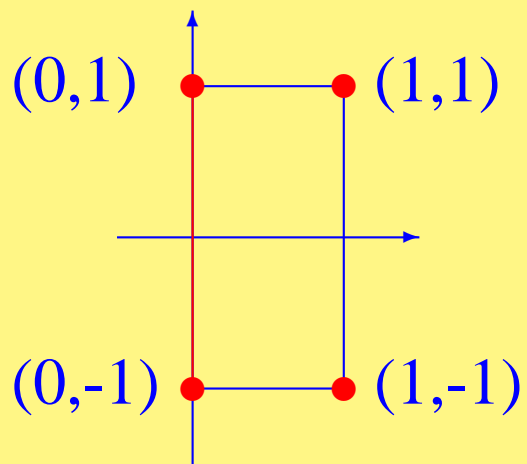
$$f(x, y) = 3x + 4y$$

$$S = \{(x, y) : \\ 0 \leq x \leq 1, \\ -1 \leq y \leq 1\}$$

$$y = 1, \quad 0 \leq x \leq 1$$

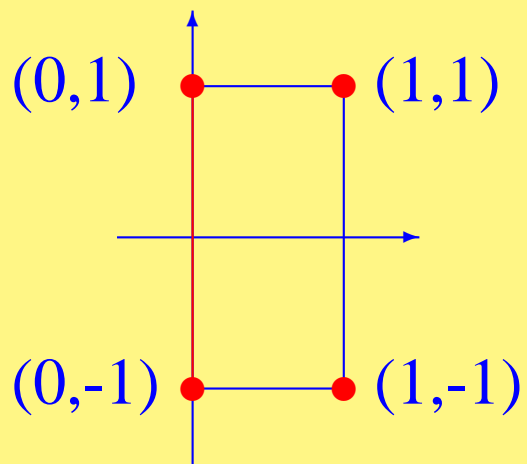
$$g(x) = 3x + 4, \quad g'(x) = 3$$

**Conclusion:** Critical points are at  $x = 0$  and  $x = 1$ .



$$f(x, y) = 3x + 4y$$

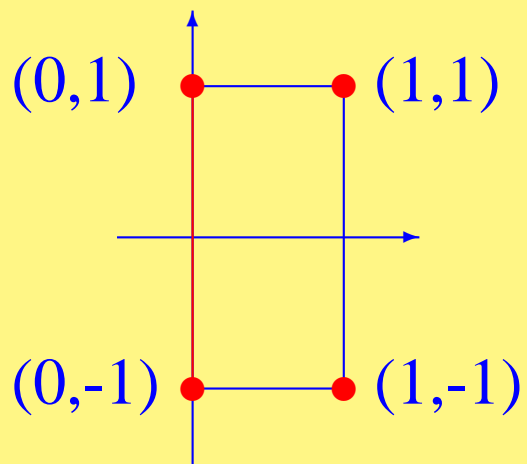
$$S = \{(x, y) : \\ 0 \leq x \leq 1, \\ -1 \leq y \leq 1\}$$



$$f(x, y) = 3x + 4y$$

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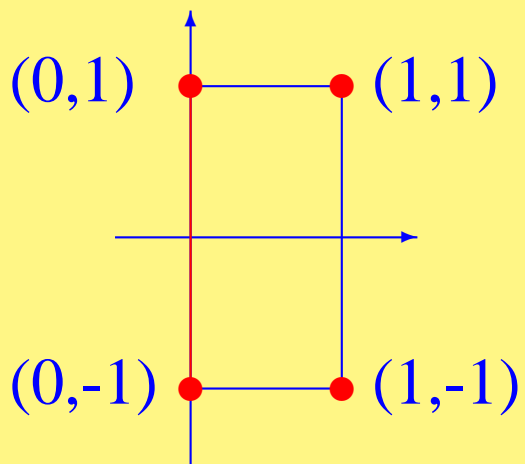
$$x = 0,$$



$$f(x, y) = 3x + 4y$$

$$S = \{(x, y) : \\ 0 \leq x \leq 1, \\ -1 \leq y \leq 1\}$$

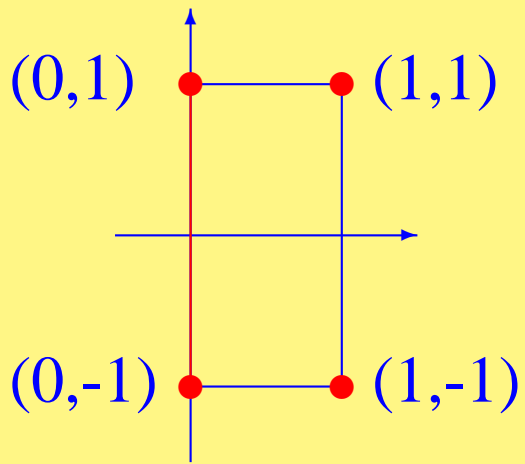
$$x = 0, \quad -1 \leq y \leq 1$$



$$f(x, y) = 3x + 4y$$

$$S = \{(x, y) : \\ 0 \leq x \leq 1, \\ -1 \leq y \leq 1\}$$

$$x = 0, \quad -1 \leq y \leq 1 \\ g(y) = 4y,$$

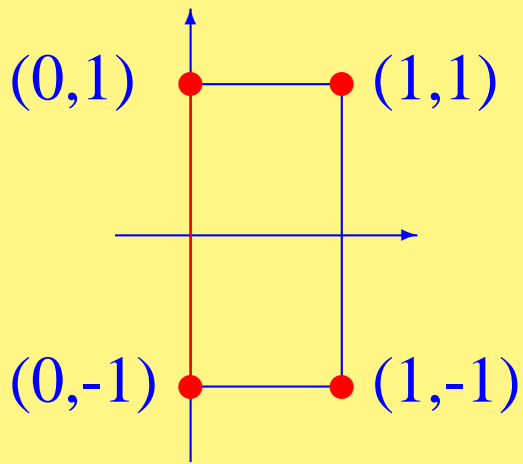


$$f(x, y) = 3x + 4y$$

$$S = \{(x, y) : \\ 0 \leq x \leq 1, \\ -1 \leq y \leq 1\}$$

$$x = 0, \quad -1 \leq y \leq 1 \\ g(y) = 4y, \quad g'(y) = 4$$



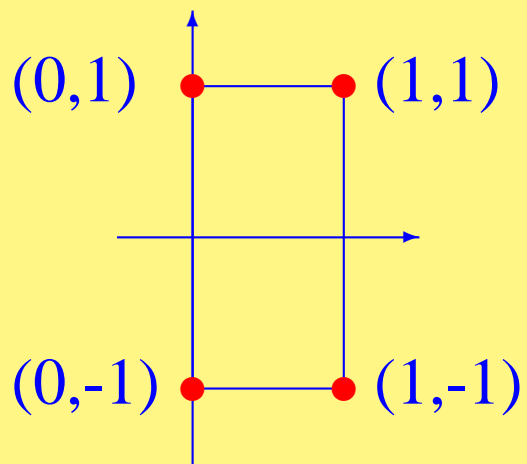


$$f(x, y) = 3x + 4y$$

$$S = \{(x, y) : \\ 0 \leq x \leq 1, \\ -1 \leq y \leq 1\}$$

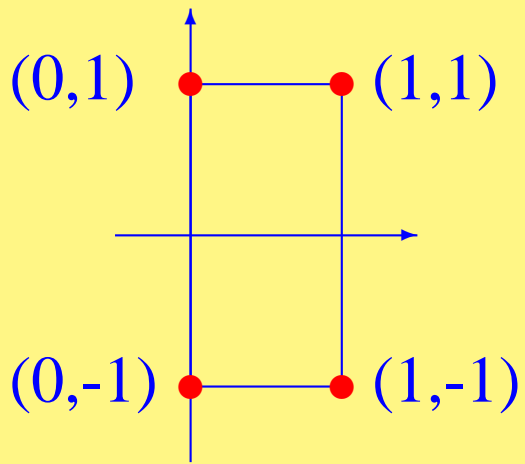
$$x = 0, \quad -1 \leq y \leq 1 \\ g(y) = 4y, \quad g'(y) = 4$$

**Conclusion:** Critical points are at  $y = -1$  and  $y = 1$ .



$$f(x, y) = 3x + 4y$$

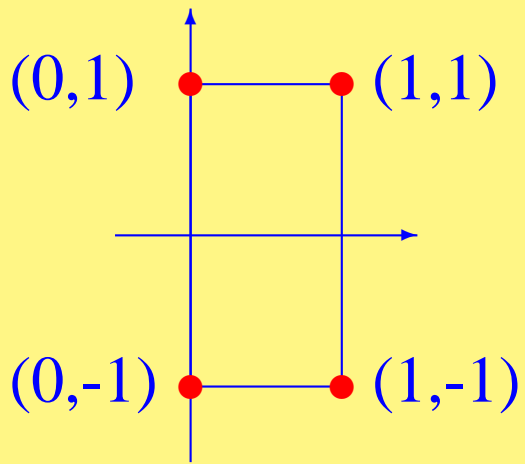
$$S = \{(x, y) : \\ 0 \leq x \leq 1, \\ -1 \leq y \leq 1\}$$



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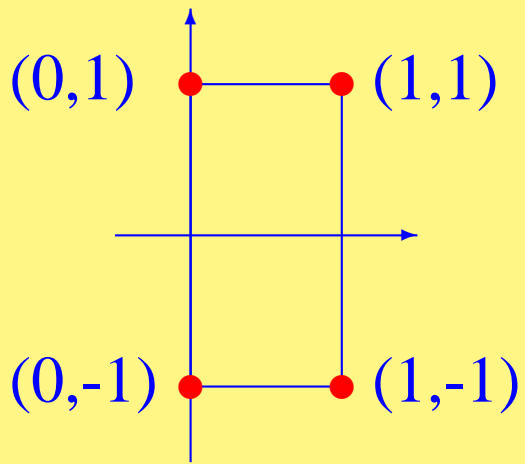
$$f(0, -1) = -4,$$



$$f(x, y) = 3x + 4y$$

$$S = \{(x, y) : \\ 0 \leq x \leq 1, \\ -1 \leq y \leq 1\}$$

$$f(0, -1) = -4, \quad f(1, -1) = -1$$



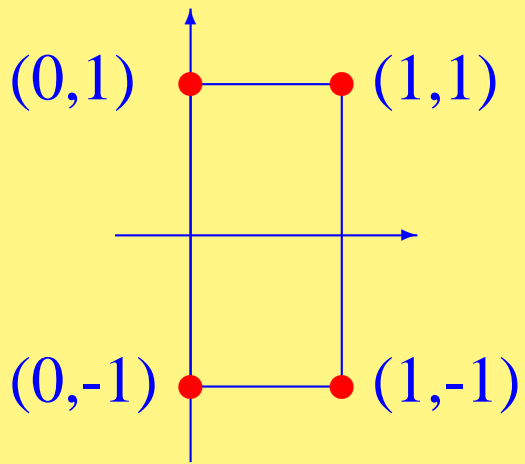
$$f(x, y) = 3x + 4y$$

$$S = \{(x, y) : \\ 0 \leq x \leq 1, \\ -1 \leq y \leq 1\}$$

$$f(0, -1) = -4,$$

$$f(1, -1) = -1$$

$$f(0, 1) = 4,$$



$$f(x, y) = 3x + 4y$$

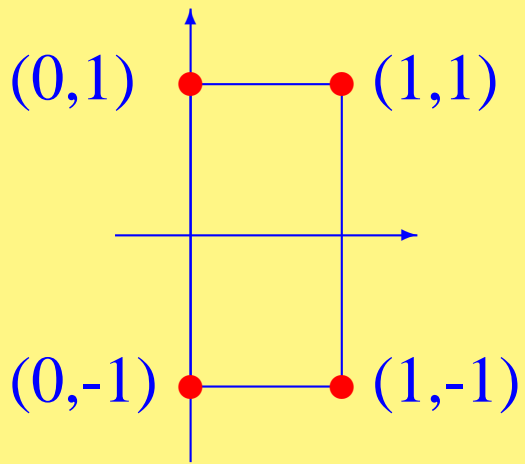
$$S = \{(x, y) : \\ 0 \leq x \leq 1, \\ -1 \leq y \leq 1\}$$

$$f(0, -1) = -4,$$

$$f(1, -1) = -1$$

$$f(0, 1) = 4,$$

$$f(1, 1) = 7$$



$$f(x, y) = 3x + 4y$$

$$S = \{(x, y) : \\ 0 \leq x \leq 1, \\ -1 \leq y \leq 1\}$$

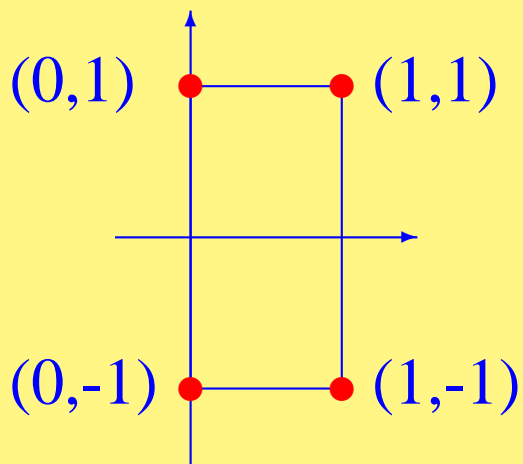
$$f(0, -1) = -4,$$

$$f(1, -1) = -1$$

$$f(0, 1) = 4,$$

$$f(1, 1) = 7$$

**Maximum:** 7 at the point  $(1, 1)$



$$f(x, y) = 3x + 4y$$

$$S = \{(x, y) : \\ 0 \leq x \leq 1, \\ -1 \leq y \leq 1\}$$

$$f(0, -1) = -4, \quad f(1, -1) = -1$$

$$f(0, 1) = 4, \quad f(1, 1) = 7$$

**Maximum:** 7 at the point (1, 1)

**Minimum:** -4 at the point (0, -1)



## Page 759, Problem 13

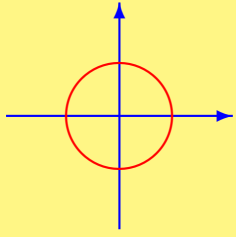
Find the maximum and minimum values of  $f$  on  $S$  where

$$f(x, y) = x^2 - y^2 + 1$$

and

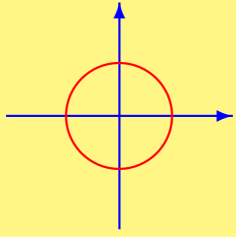
$$S = \{(x, y) : x^2 + y^2 \leq 1\}$$

and indicate where they occur.



$$f(x, y) = x^2 - y^2 + 1$$

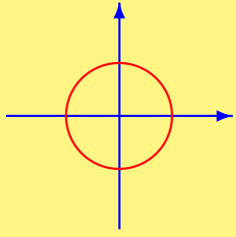
$$S = \{(x, y) : \\ x^2 + y^2 \leq 1\}$$



$$f(x, y) = x^2 - y^2 + 1$$

$$S = \{(x, y) : \\ x^2 + y^2 \leq 1\}$$

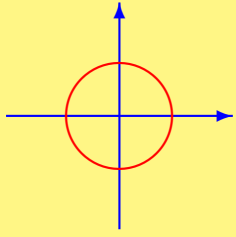
$$\nabla f =$$



$$f(x, y) = x^2 - y^2 + 1$$

$$S = \{(x, y) : \\ x^2 + y^2 \leq 1\}$$

$$\nabla f = \langle 2x, -2y \rangle$$

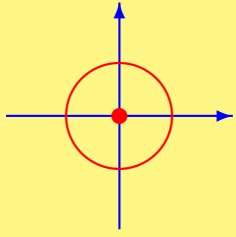


$$f(x, y) = x^2 - y^2 + 1$$

$$S = \{(x, y) : \\ x^2 + y^2 \leq 1\}$$

$$\nabla f = \langle 2x, -2y \rangle$$

**Conclusion:** Critical points are  $(0, 0)$  and points on the boundary.

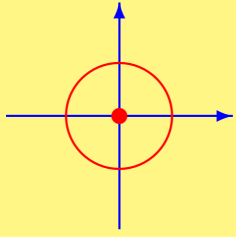


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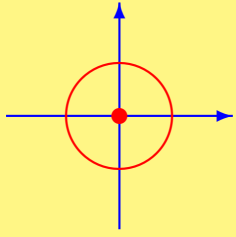
$$\nabla f = \langle 2x, -2y \rangle$$

**Conclusion:** Critical points are  $(0, 0)$  and points on the boundary.



$$f(x, y) = x^2 - y^2 + 1$$

$$S = \{(x, y) : \\ x^2 + y^2 \leq 1\}$$

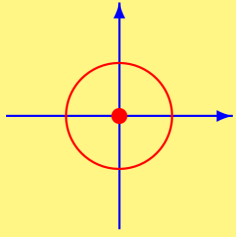


$$f(x, y) = x^2 - y^2 + 1$$

$$S = \{(x, y) : \\ x^2 + y^2 \leq 1\}$$

**On Boundary:**

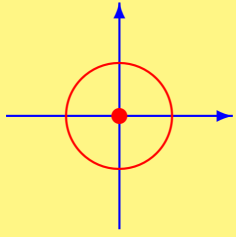




$$f(x, y) = x^2 - y^2 + 1$$

$$S = \{(x, y) : \\ x^2 + y^2 \leq 1\}$$

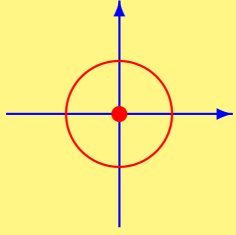
**On Boundary:**  $y^2 = 1 - x^2$



$$f(x, y) = x^2 - y^2 + 1$$

$$S = \{(x, y) : \\ x^2 + y^2 \leq 1\}$$

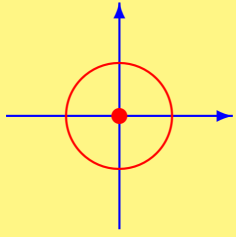
**On Boundary:**  $y^2 = 1 - x^2$  so  $g(x) =$



$$f(x, y) = x^2 - y^2 + 1$$

$$S = \{(x, y) : x^2 + y^2 \leq 1\}$$

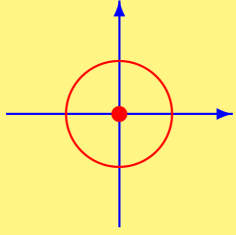
**On Boundary:**  $y^2 = 1 - x^2$  so  $g(x) = 2x^2$



$$f(x, y) = x^2 - y^2 + 1$$

$$S = \{(x, y) : \\ x^2 + y^2 \leq 1\}$$

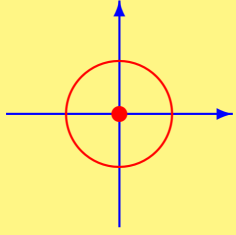
**On Boundary:**  $y^2 = 1 - x^2$  so  $g(x) = 2x^2$   
 $-1 \leq x \leq 1,$



$$f(x, y) = x^2 - y^2 + 1$$

$$S = \{(x, y) : x^2 + y^2 \leq 1\}$$

**On Boundary:**  $y^2 = 1 - x^2$  so  $g(x) = 2x^2$   
 $-1 \leq x \leq 1, \quad g'(x) = 4x$

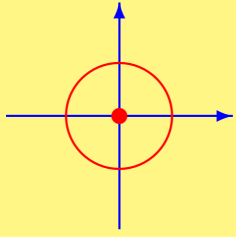


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**Points On Boundary To Consider:**

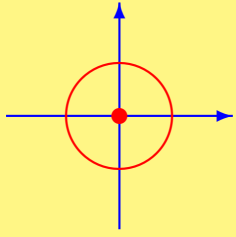


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**Points On Boundary To Consider:**  $(0, \pm 1)$



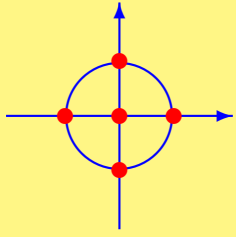
$$f(x, y) = x^2 - y^2 + 1$$

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**On Boundary:**  $y^2 = 1 - x^2$  so  $g(x) = 2x^2$   
 $-1 \leq x \leq 1, \quad g'(x) = 4x$

**Points On Boundary To Consider:**  $(0, \pm 1), (\pm 1, 0)$



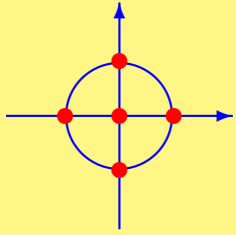


$$f(x, y) = x^2 - y^2 + 1$$

$$S = \{(x, y) : x^2 + y^2 \leq 1\}$$

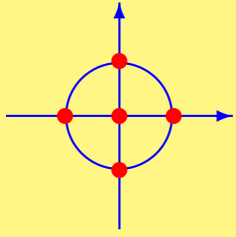
**On Boundary:**  $y^2 = 1 - x^2$  so  $g(x) = 2x^2$   
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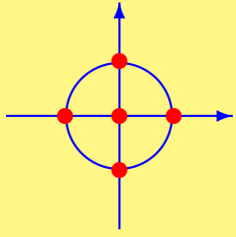
$$S = \{(x, y) : \\ x^2 + y^2 \leq 1\}$$



$$f(x, y) = x^2 - y^2 + 1$$

$$S = \{(x, y) : x^2 + y^2 \leq 1\}$$

**Points To Consider:**  $(0, 0)$ ,  $(0, \pm 1)$ ,  $(\pm 1, 0)$

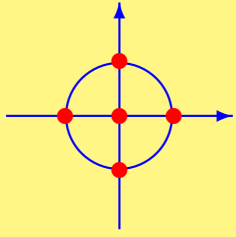


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$$S = \{(x, y) : x^2 + y^2 \leq 1\}$$

**Points To Consider:**  $(0, 0)$ ,  $(0, \pm 1)$ ,  $(\pm 1, 0)$

$$f(0, 0) = 1,$$

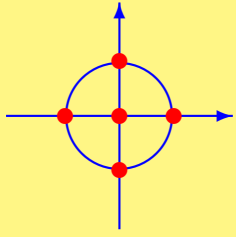


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**Points To Consider:**  $(0, 0)$ ,  $(0, \pm 1)$ ,  $(\pm 1, 0)$

$$f(0, 0) = 1, \quad f(0, \pm 1) = 0,$$

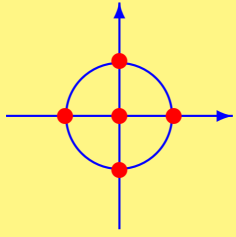


$$f(x, y) = x^2 - y^2 + 1$$

$$S = \{(x, y) : x^2 + y^2 \leq 1\}$$

**Points To Consider:**  $(0, 0)$ ,  $(0, \pm 1)$ ,  $(\pm 1, 0)$

$$f(0, 0) = 1, \quad f(0, \pm 1) = 0, \quad f(\pm 1, 0) = 2$$



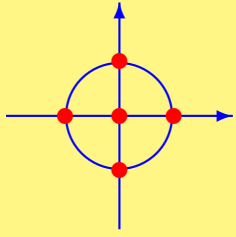
$$f(x, y) = x^2 - y^2 + 1$$

$$S = \{(x, y) : x^2 + y^2 \leq 1\}$$

**Points To Consider:**  $(0, 0)$ ,  $(0, \pm 1)$ ,  $(\pm 1, 0)$

$$f(0, 0) = 1, \quad f(0, \pm 1) = 0, \quad f(\pm 1, 0) = 2$$

**Maximum:**  $2$  at the point  $(\pm 1, 0)$



$$f(x, y) = x^2 - y^2 + 1$$

$$S = \{(x, y) : x^2 + y^2 \leq 1\}$$

**Points To Consider:**  $(0, 0)$ ,  $(0, \pm 1)$ ,  $(\pm 1, 0)$

$$f(0, 0) = 1, \quad f(0, \pm 1) = 0, \quad f(\pm 1, 0) = 2$$

**Maximum:**  $2$  at the point  $(\pm 1, 0)$

**Minimum:**  $0$  at the point  $(0, \pm 1)$