Cross Product Formula

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$$\boxed{|\vec{u} \times \vec{v}| = |\vec{u}|\,|\vec{v}| \sin\theta}$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 \, |\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$$

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$$\vec{u} \times \vec{v} \quad = \det \begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & b & c \\ d & e & f \end{pmatrix}$$

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$$\begin{aligned} \vec{u} \times \vec{v} &= \det \begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & b & c \\ d & e & f \end{pmatrix} \\ &= \langle bf \end{aligned}$$

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$$\begin{aligned} \vec{u} \times \vec{v} &= \det \begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & b & c \\ d & e & f \end{pmatrix} \\ &= \langle bf - ce, \end{aligned}$$

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$$\begin{aligned} \vec{u} \times \vec{v} &= \det \begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & b & c \\ d & e & f \end{pmatrix} \\ &= \langle bf - ce, cd \end{aligned}$$

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$$\begin{aligned} |\vec{u} \times \vec{v}|^2 &= |\langle bf - ce, cd - af, ae - bd \rangle|^2 \\ &= (bf - ce)^2 + (cd - af)^2 + (ae - bd)^2 \end{aligned}$$

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$$(\vec{u} \cdot \vec{v})^2$$

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$$|\vec{u} \times \vec{v}|^2 = (bf - ce)^2 + (cd - af)^2 + (ae - bd)^2$$

$$(\vec{u} \cdot \vec{v})^2 = (ad + be + cf)^2$$

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$$\begin{aligned} (\vec{u} \cdot \vec{v})^2 &= (ad + be + cf)(ad + be + cf) \\ &= a^2 d^2 \end{aligned}$$

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$$\begin{aligned} (\vec{u} \cdot \vec{v})^2 &= (ad + be + cf)(ad + be + cf) \\ &= a^2d^2 + b^2e^2 + c^2f^2 \\ &\quad + 2adbe \end{aligned}$$

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$$b^2 f^2 + c^2 e^2 + c^2 d^2 + a^2 f^2 + a^2 e^2 + b^2 d^2 + a^2 d^2 + b^2 e^2 + c^2 f^2$$

$$= a^2 (d^2 + e^2 + f^2)$$

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$$|\vec{u} \times \vec{v}|^2 = (bf - ce)^2 + (cd - af)^2 + (ae - bd)^2$$

$$(\vec{u} \cdot \vec{v})^2 = a^2 d^2 + b^2 e^2 + c^2 f^2 + 2adbe + 2adcf + 2becf$$

$$b^2 f^2 + c^2 e^2 + c^2 d^2 + a^2 f^2 + a^2 e^2 + b^2 d^2 + a^2 d^2 + b^2 e^2 + c^2 f^2$$

$$= a^2 (d^2 + e^2 + f^2)$$

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$$|\vec{u} \times \vec{v}|^2 = (bf - ce)^2 + (cd - af)^2 + (ae - bd)^2$$

$$(\vec{u} \cdot \vec{v})^2 = a^2 d^2 + b^2 e^2 + c^2 f^2 + 2adbe + 2adcf + 2becf$$

$$b^2 f^2 + c^2 e^2 + c^2 d^2 + a^2 f^2 + a^2 e^2 + b^2 d^2 + a^2 d^2 + b^2 e^2 + c^2 f^2$$

$$= (a^2 + b^2)(d^2 + e^2 + f^2)$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 \, |\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$$

$$\vec{u} = \langle a, b, c \rangle \qquad \vec{v} = \langle d, e, f \rangle$$

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$$b^2 f^2 + c^2 e^2 + c^2 d^2 + a^2 f^2 + a^2 e^2 + b^2 d^2 + a^2 d^2 + b^2 e^2 + c^2 f^2$$

$$= (a^2 + b^2 + c^2)(d^2 + e^2 + f^2)$$

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$$(\vec{u} \cdot \vec{v})^2 = a^2 d^2 + b^2 e^2 + c^2 f^2 + 2adbe + 2adcf + 2becf$$

$$b^2 f^2 + c^2 e^2 + c^2 d^2 + a^2 f^2 + a^2 e^2 + b^2 d^2 + a^2 d^2 + b^2 e^2 + c^2 f^2$$

$$= |\vec{u}|^2 (d^2 + e^2 + f^2)$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 \, |\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$$

$$\vec{u} = \langle a, b, c \rangle \qquad \vec{v} = \langle d, e, f \rangle$$

$$|\vec{u} \times \vec{v}|^2 = (bf - ce)^2 + (cd - af)^2 + (ae - bd)^2$$

$$(\vec{u} \cdot \vec{v})^2 = a^2 d^2 + b^2 e^2 + c^2 f^2 + 2adbe + 2adcf + 2becf$$

$$b^2 f^2 + c^2 e^2 + c^2 d^2 + a^2 f^2 + a^2 e^2 + b^2 d^2 + a^2 d^2 + b^2 e^2 + c^2 f^2$$

$$= \quad |\vec{u}|^2 \qquad |\vec{v}|^2$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 \, |\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$$

$$\vec{u} = \langle a, b, c \rangle \qquad \vec{v} = \langle d, e, f \rangle$$

$$|\vec{u} \times \vec{v}|^2 + (\vec{u} \cdot \vec{v})^2 = |\vec{u}|^2 \, |\vec{v}|^2$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 \, |\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 \, |\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$$

$$\downarrow$$

$$(|\vec{u}| \, |\vec{v}| \cos\theta)^2$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 \, |\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 \, |\vec{v}|^2 - (|\vec{u}| \, |\vec{v}| \cos\theta)^2$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 \, |\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$$

$$\begin{aligned} |\vec{u} \times \vec{v}|^2 &= |\vec{u}|^2 \, |\vec{v}|^2 - (|\vec{u}| \, |\vec{v}| \cos\theta)^2 \\ &= |\vec{u}|^2 \, |\vec{v}|^2 - |\vec{u}|^2 |\vec{v}|^2 \cos^2\theta \end{aligned}$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 \, |\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$$

$$\begin{aligned} |\vec{u} \times \vec{v}|^2 &= |\vec{u}|^2 \, |\vec{v}|^2 - (|\vec{u}| \, |\vec{v}| \cos\theta)^2 \\ &= |\vec{u}|^2 \, |\vec{v}|^2 - |\vec{u}|^2 |\vec{v}|^2 \cos^2\theta \\ &= |\vec{u}|^2 \, |\vec{v}|^2 (1 - \cos^2\theta) \end{aligned}$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 \, |\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$$

$$\begin{aligned}
|\vec{u} \times \vec{v}|^2 &= |\vec{u}|^2 \, |\vec{v}|^2 - (|\vec{u}| \, |\vec{v}| \cos\theta)^2 \\
&= |\vec{u}|^2 \, |\vec{v}|^2 - |\vec{u}|^2 |\vec{v}|^2 \cos^2\theta \\
&= |\vec{u}|^2 \, |\vec{v}|^2 (1 - \cos^2\theta) \\
&= |\vec{u}|^2 \, |\vec{v}|^2 \sin^2\theta
\end{aligned}$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 \, |\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 \, |\vec{v}|^2 \sin^2 \theta$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 \, |\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 \, |\vec{v}|^2 \sin^2 \theta$$

$$0 \leq \theta \leq \pi$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 \, |\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 \, |\vec{v}|^2 \sin^2 \theta$$

$$0 \leq \theta \leq \pi$$

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