

Cross Product Formula

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$$|\vec{u} \times \vec{v}| = |\vec{u}| |\vec{v}| \sin \theta$$

$$|\vec{u}\times\vec{v}|^2=|\vec{u}|^2\,|\vec{v}|^2-(\vec{u}\cdot\vec{v})^2$$

$$\begin{aligned}|\vec{u}\times\vec{v}|^2&=|\vec{u}|^2\,|\vec{v}|^2-(\vec{u}\cdot\vec{v})^2\\\vec{u}&=\langle a,b,c\rangle\qquad\vec{v}=\langle d,e,f\rangle\end{aligned}$$

$$\begin{aligned}|\vec{u}\times\vec{v}|^2&=|\vec{u}|^2\,|\vec{v}|^2-(\vec{u}\cdot\vec{v})^2\\\vec{u}&=\langle a,b,c\rangle\qquad\vec{v}=\langle d,e,f\rangle\end{aligned}$$

$$\vec{u}\times\vec{v}=\det\begin{pmatrix}\vec{i}&\vec{j}&\vec{k}\\a&b&c\\d&e&f\end{pmatrix}$$

$$|\vec{u}\times\vec{v}|^2=|\vec{u}|^2\,|\vec{v}|^2-(\vec{u}\cdot\vec{v})^2$$

$$\vec{u}=\langle a,b,c\rangle \qquad \vec{v}=\langle d,e,f\rangle$$

$$\begin{aligned}\vec{u}\times\vec{v}&=\det\begin{pmatrix}\vec{i}&\vec{j}&\vec{k}\\a&\textcolor{red}{b}&c\\d&e&\textcolor{red}{f}\end{pmatrix}\\&=\langle bf\end{aligned}$$

$$|\vec{u}\times\vec{v}|^2=|\vec{u}|^2\,|\vec{v}|^2-(\vec{u}\cdot\vec{v})^2$$

$$\vec{u}=\langle a,b,c\rangle \qquad \vec{v}=\langle d,e,f\rangle$$

$$\begin{aligned}\vec{u}\times\vec{v}&=\det\begin{pmatrix}\vec{i}&\vec{j}&\vec{k}\\a&b&\textcolor{red}{c}\\d&\textcolor{red}{e}&f\end{pmatrix}\\&=\langle bf-ce,\end{aligned}$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2\,|\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$$

$$\vec{u}=\langle a,b,c\rangle \qquad \vec{v}=\langle d,e,f\rangle$$

$$\begin{aligned}\vec{u}\times\vec{v}&=\det\begin{pmatrix}\vec{i}&\vec{j}&\vec{k}\\a&b&\textcolor{red}{c}\\ \textcolor{red}{d}&e&f\end{pmatrix}\\&=\langle bf-ce,cd\end{aligned}$$

$$|\vec{u}\times\vec{v}|^2=|\vec{u}|^2\,|\vec{v}|^2-(\vec{u}\cdot\vec{v})^2$$

$$\vec{u}=\langle a,b,c\rangle \qquad \vec{v}=\langle d,e,f\rangle$$

$$\begin{aligned}\vec{u}\times\vec{v}&=\det\begin{pmatrix}\vec{i}&\vec{j}&\vec{k}\\\textcolor{red}{a}&b&c\\d&e&\textcolor{red}{f}\end{pmatrix}\\&=\langle bf-ce,cd-af,$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2\,|\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$$

$$\vec{u}=\langle a,b,c\rangle \qquad \vec{v}=\langle d,e,f\rangle$$

$$\begin{aligned}\vec{u} \times \vec{v} &= \det \begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ \textcolor{red}{a} & b & c \\ d & \textcolor{red}{e} & f \end{pmatrix} \\ &= \langle bf-ce, cd-af, ae \end{aligned}$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2\,|\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$$

$$\vec{u}=\langle a,b,c\rangle \qquad \vec{v}=\langle d,e,f\rangle$$

$$\begin{aligned}\vec{u} \times \vec{v} &= \det \begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & \textcolor{red}{b} & c \\ \textcolor{red}{d} & e & f \end{pmatrix} \\ &= \langle bf-ce, cd-af, ae-bd \rangle\end{aligned}$$

$$|\vec{u}\times\vec{v}|^2=|\vec{u}|^2\,|\vec{v}|^2-(\vec{u}\cdot\vec{v})^2$$

$$\vec{u}=\langle a,b,c\rangle \qquad \vec{v}=\langle d,e,f\rangle$$

$$\begin{aligned}\vec{u}\times\vec{v}&=\det\begin{pmatrix}\vec{i}&\vec{j}&\vec{k}\\a&b&c\\d&e&f\end{pmatrix}\\&=\langle bf-ce,cd-af,ae-bd\rangle\end{aligned}$$

$$|\vec{u}\times\vec{v}|^2=|\vec{u}|^2\,|\vec{v}|^2-(\vec{u}\cdot\vec{v})^2$$

$$\vec{u}=\langle a,b,c\rangle \qquad \vec{v}=\langle d,e,f\rangle$$

$$\vec{u}\times\vec{v}~=~\langle bf-ce,cd-af,ae-bd\rangle$$

$$\begin{aligned}|\vec{u}\times\vec{v}|^2&=|\vec{u}|^2\,|\vec{v}|^2-(\vec{u}\cdot\vec{v})^2\\\vec{u}&=\langle a,b,c\rangle\qquad\vec{v}=\langle d,e,f\rangle\end{aligned}$$

$$|\vec{u}\times\vec{v}|^2=|\langle bf-ce,cd-af,ae-bd\rangle|^2$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2\,|\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$$

$$\vec{u}=\langle a,b,c\rangle \qquad \vec{v}=\langle d,e,f\rangle$$

$$\begin{aligned}|\vec{u} \times \vec{v}|^2 &= |\langle bf-ce, cd-af, ae-bd \rangle|^2 \\&= (bf-ce)^2 + (cd-af)^2 + (ae-bd)^2\end{aligned}$$

$$\begin{aligned}|\vec{u}\times\vec{v}|^2&=|\vec{u}|^2\,|\vec{v}|^2-(\vec{u}\cdot\vec{v})^2\\\vec{u}&=\langle a,b,c\rangle\qquad\vec{v}=\langle d,e,f\rangle\end{aligned}$$

$$|\vec{u}\times\vec{v}|^2=(bf-ce)^2+(cd-af)^2+(ae-bd)^2$$

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$$|\vec{u}\times\vec{v}|^2=(bf-ce)^2+(cd-af)^2+(ae-bd)^2$$

$$(\vec{u}\cdot\vec{v})^2$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2\,|\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$$

$$\vec{u}=\langle a,b,c\rangle \qquad \vec{v}=\langle d,e,f\rangle$$

$$|\vec{u} \times \vec{v}|^2 = (bf-ce)^2 + (cd-af)^2 + (ae-bd)^2$$

$$(\vec{u}\cdot\vec{v})^2=(ad+be+cf)^2$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2\,|\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$$

$$\vec{u}=\langle a,b,c\rangle \qquad \vec{v}=\langle d,e,f\rangle$$

$$|\vec{u} \times \vec{v}|^2 = (bf-ce)^2 + (cd-af)^2 + (ae-bd)^2$$

$$(\vec{u}\cdot\vec{v})^2=(ad+be+cf)(ad+be+cf)$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 |\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$$

$$\vec{u} = \langle a, b, c \rangle \quad \quad \vec{v} = \langle d, e, f \rangle$$

$$|\vec{u} \times \vec{v}|^2 = (bf-ce)^2 + (cd-af)^2 + (ae-bd)^2$$

$$\begin{aligned}(\vec{u} \cdot \vec{v})^2 &= (\textcolor{red}{ad} + be + cf)(\textcolor{red}{ad} + be + cf) \\&= a^2d^2\end{aligned}$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 |\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$$

$$\vec{u} = \langle a, b, c \rangle \quad \quad \vec{v} = \langle d, e, f \rangle$$

$$|\vec{u} \times \vec{v}|^2 = (bf-ce)^2 + (cd-af)^2 + (ae-bd)^2$$

$$\begin{aligned}(\vec{u} \cdot \vec{v})^2 &= (ad + \textcolor{red}{be} + cf)(ad + \textcolor{red}{be} + cf) \\&= a^2d^2 + b^2e^2\end{aligned}$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 \, |\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$$

$$\vec{u} = \langle a,b,c \rangle \qquad \vec{v} = \langle d,e,f \rangle$$

$$|\vec{u} \times \vec{v}|^2 = (bf-ce)^2 + (cd-af)^2 + (ae-bd)^2$$

$$\begin{aligned}(\vec{u} \cdot \vec{v})^2 &= (ad+be+\textcolor{red}{cf})(ad+be+\textcolor{red}{cf}) \\&= a^2d^2+b^2e^2+c^2f^2\end{aligned}$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 |\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$$

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$$\begin{aligned}(\vec{u} \cdot \vec{v})^2 &= (\textcolor{red}{ad} + be + cf)(ad + \textcolor{red}{be} + cf) \\&= a^2d^2 + b^2e^2 + c^2f^2\end{aligned}$$

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$$|\vec{u} \times \vec{v}|^2 = (bf-ce)^2 + (cd-af)^2 + (ae-bd)^2$$

$$\begin{aligned}(\vec{u} \cdot \vec{v})^2 &= (\textcolor{red}{ad} + \textcolor{violet}{be} + cf)(ad + \textcolor{red}{be} + cf) \\&= a^2d^2 + b^2e^2 + c^2f^2\end{aligned}$$

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$$|\vec{u} \times \vec{v}|^2 = (bf - ce)^2 + (cd - af)^2 + (ae - bd)^2$$

$$\begin{aligned}(\vec{u} \cdot \vec{v})^2 &= (\textcolor{red}{ad} + \textcolor{violet}{be} + cf)(ad + \textcolor{red}{be} + cf) \\&= a^2d^2 + b^2e^2 + c^2f^2 \\&\quad + 2adbe\end{aligned}$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 |\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$$

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$$|\vec{u} \times \vec{v}|^2 = (bf-ce)^2 + (cd-af)^2 + (ae-bd)^2$$

$$\begin{aligned}(\vec{u} \cdot \vec{v})^2 &= (\textcolor{red}{ad} + be + \textcolor{magenta}{cf})(ad + be + \textcolor{red}{cf}) \\&= a^2d^2 + b^2e^2 + c^2f^2 \\&\quad + 2adbe + 2adcf\end{aligned}$$

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$$|\vec{u} \times \vec{v}|^2 = (bf-ce)^2 + (cd-af)^2 + (ae-bd)^2$$

$$\begin{aligned}(\vec{u} \cdot \vec{v})^2 &= (ad + \textcolor{red}{be} + \textcolor{pink}{cf})(ad + \textcolor{red}{be} + \textcolor{pink}{cf}) \\&= a^2d^2 + b^2e^2 + c^2f^2 \\&\quad + 2adbe + 2adcf + 2becf\end{aligned}$$

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$$\vec{u} = \langle a, b, c \rangle \qquad \vec{v} = \langle d, e, f \rangle$$

$$|\vec{u} \times \vec{v}|^2 = (bf-ce)^2 + (cd-af)^2 + (ae-bd)^2$$

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$$|\vec{u} \times \vec{v}|^2 = (\textcolor{red}{bf-ce})^2 + (cd-af)^2 + (ae-bd)^2$$

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$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 |\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$$

$$\vec{u} = \langle a, b, c \rangle \quad \quad \vec{v} = \langle d, e, f \rangle$$

$$|\vec{u} \times \vec{v}|^2 = (\textcolor{red}{bf} - \textcolor{blue}{ce})^2 + (cd - af)^2 + (ae - bd)^2$$

$$(\vec{u} \cdot \vec{v})^2 = a^2d^2 + b^2e^2 + c^2f^2$$

$$+ 2adbe + 2adcf + \textcolor{red}{2becf}$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 |\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$$

$$\vec{u} = \langle a, b, c \rangle \quad \quad \vec{v} = \langle d, e, f \rangle$$

$$|\vec{u} \times \vec{v}|^2 = (bf-ce)^2 + (\textcolor{red}{cd-af})^2 + (ae-bd)^2$$

$$(\vec{u} \cdot \vec{v})^2 = a^2d^2 + b^2e^2 + c^2f^2$$

$$+ 2adbe + 2adcf + 2becf$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 |\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$$

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$$|\vec{u} \times \vec{v}|^2 = (bf - ce)^2 + (cd - af)^2 + (ae - bd)^2$$

$$(\vec{u} \cdot \vec{v})^2 = a^2d^2 + b^2e^2 + c^2f^2$$

$$+ 2adbe + 2adcf + 2becf$$

$$b^2f^2 + c^2e^2 + c^2d^2 + a^2f^2 + a^2e^2 + b^2d^2 \\ + a^2d^2 + b^2e^2 + c^2f^2$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 |\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$$

$$\vec{u} = \langle a, b, c \rangle \quad \vec{v} = \langle d, e, f \rangle$$

$$|\vec{u} \times \vec{v}|^2 = (bf - ce)^2 + (cd - af)^2 + (ae - bd)^2$$

$$(\vec{u} \cdot \vec{v})^2 = a^2d^2 + b^2e^2 + c^2f^2$$

$$+ 2adbe + 2adcf + 2becf$$

$$\begin{aligned} & b^2f^2 + c^2e^2 + c^2d^2 + \textcolor{red}{a^2f^2} + \textcolor{red}{a^2e^2} + b^2d^2 \\ & + \textcolor{red}{a^2d^2} + b^2e^2 + c^2f^2 \end{aligned}$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 |\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$$

$$\vec{u} = \langle a, b, c \rangle \quad \vec{v} = \langle d, e, f \rangle$$

$$|\vec{u} \times \vec{v}|^2 = (bf - ce)^2 + (cd - af)^2 + (ae - bd)^2$$

$$(\vec{u} \cdot \vec{v})^2 = a^2d^2 + b^2e^2 + c^2f^2$$

$$+ 2adbe + 2adcf + 2becf$$

$$\begin{aligned} & b^2f^2 + c^2e^2 + c^2d^2 + \textcolor{red}{a^2f^2} + \textcolor{red}{a^2e^2} + b^2d^2 \\ & + \textcolor{red}{a^2d^2} + b^2e^2 + c^2f^2 \\ & = \textcolor{red}{a^2(d^2 + e^2 + f^2)} \end{aligned}$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 |\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$$

$$\vec{u} = \langle a, b, c \rangle \quad \vec{v} = \langle d, e, f \rangle$$

$$|\vec{u} \times \vec{v}|^2 = (bf - ce)^2 + (cd - af)^2 + (ae - bd)^2$$

$$(\vec{u} \cdot \vec{v})^2 = a^2d^2 + b^2e^2 + c^2f^2$$

$$+ 2adbe + 2adcf + 2becf$$

$$\begin{aligned} & b^2f^2 + c^2e^2 + c^2d^2 + a^2f^2 + a^2e^2 + b^2d^2 \\ & + a^2d^2 + b^2e^2 + c^2f^2 \\ & = a^2(d^2 + e^2 + f^2) \end{aligned}$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 |\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$$

$$\vec{u} = \langle a, b, c \rangle \quad \vec{v} = \langle d, e, f \rangle$$

$$|\vec{u} \times \vec{v}|^2 = (bf - ce)^2 + (cd - af)^2 + (ae - bd)^2$$

$$(\vec{u} \cdot \vec{v})^2 = a^2d^2 + b^2e^2 + c^2f^2$$

$$+ 2adbe + 2adcf + 2becf$$

$$\begin{aligned} & b^2f^2 + c^2e^2 + c^2d^2 + a^2f^2 + a^2e^2 + b^2d^2 \\ & + a^2d^2 + b^2e^2 + c^2f^2 \\ & = (a^2 + b^2)(d^2 + e^2 + f^2) \end{aligned}$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 |\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$$

$$\vec{u} = \langle a, b, c \rangle \quad \vec{v} = \langle d, e, f \rangle$$

$$|\vec{u} \times \vec{v}|^2 = (bf - ce)^2 + (cd - af)^2 + (ae - bd)^2$$

$$(\vec{u} \cdot \vec{v})^2 = a^2d^2 + b^2e^2 + c^2f^2$$

$$+ 2adbe + 2adcf + 2becf$$

$$\begin{aligned} & b^2f^2 + c^2e^2 + c^2d^2 + a^2f^2 + a^2e^2 + b^2d^2 \\ & + a^2d^2 + b^2e^2 + c^2f^2 \\ & = (a^2 + b^2 + c^2)(d^2 + e^2 + f^2) \end{aligned}$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 |\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$$

$$\vec{u} = \langle a, b, c \rangle \quad \vec{v} = \langle d, e, f \rangle$$

$$|\vec{u} \times \vec{v}|^2 = (bf - ce)^2 + (cd - af)^2 + (ae - bd)^2$$

$$(\vec{u} \cdot \vec{v})^2 = a^2d^2 + b^2e^2 + c^2f^2$$

$$+ 2adbe + 2adcf + 2becf$$

$$\begin{aligned} & b^2f^2 + c^2e^2 + c^2d^2 + a^2f^2 + a^2e^2 + b^2d^2 \\ & + a^2d^2 + b^2e^2 + c^2f^2 \\ = & \quad |\vec{u}|^2 \quad (d^2 + e^2 + f^2) \end{aligned}$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 |\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$$

$$\vec{u} = \langle a, b, c \rangle \quad \vec{v} = \langle d, e, f \rangle$$

$$|\vec{u} \times \vec{v}|^2 = (bf - ce)^2 + (cd - af)^2 + (ae - bd)^2$$

$$(\vec{u} \cdot \vec{v})^2 = a^2d^2 + b^2e^2 + c^2f^2$$

$$+ 2adbe + 2adcf + 2becf$$

$$\begin{aligned} & b^2f^2 + c^2e^2 + c^2d^2 + a^2f^2 + a^2e^2 + b^2d^2 \\ & + a^2d^2 + b^2e^2 + c^2f^2 \\ = & \quad |\vec{u}|^2 \quad \quad \quad |\vec{v}|^2 \end{aligned}$$

$$|\vec{u}\times\vec{v}|^2=|\vec{u}|^2\,|\vec{v}|^2-(\vec{u}\cdot\vec{v})^2$$

$$\vec{u}=\langle a,b,c\rangle \qquad \vec{v}=\langle d,e,f\rangle$$

$$|\vec{u}\times\vec{v}|^2+(\vec{u}\cdot\vec{v})^2=|\vec{u}|^2\,|\vec{v}|^2$$

$$|\vec{u}\times\vec{v}|^2=|\vec{u}|^2\,|\vec{v}|^2-(\vec{u}\cdot\vec{v})^2$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 |\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$$



$$(|\vec{u}| |\vec{v}| \cos \theta)^2$$

$$|\vec{u}\times\vec{v}|^2=|\vec{u}|^2\,|\vec{v}|^2-(\vec{u}\cdot\vec{v})^2$$

$$|\vec{u}\times\vec{v}|^2=|\vec{u}|^2\,|\vec{v}|^2-(|\vec{u}|\,|\vec{v}|\cos\theta)^2$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 |\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$$

$$\begin{aligned} |\vec{u} \times \vec{v}|^2 &= |\vec{u}|^2 |\vec{v}|^2 - (|\vec{u}| |\vec{v}| \cos \theta)^2 \\ &= |\vec{u}|^2 |\vec{v}|^2 - |\vec{u}|^2 |\vec{v}|^2 \cos^2 \theta \end{aligned}$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 |\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$$

$$\begin{aligned} |\vec{u} \times \vec{v}|^2 &= |\vec{u}|^2 |\vec{v}|^2 - (|\vec{u}| |\vec{v}| \cos \theta)^2 \\ &= |\vec{u}|^2 |\vec{v}|^2 - |\vec{u}|^2 |\vec{v}|^2 \cos^2 \theta \\ &= |\vec{u}|^2 |\vec{v}|^2 (1 - \cos^2 \theta) \end{aligned}$$

$$|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 |\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$$

$$\begin{aligned} |\vec{u} \times \vec{v}|^2 &= |\vec{u}|^2 |\vec{v}|^2 - (|\vec{u}| |\vec{v}| \cos \theta)^2 \\ &= |\vec{u}|^2 |\vec{v}|^2 - |\vec{u}|^2 |\vec{v}|^2 \cos^2 \theta \\ &= |\vec{u}|^2 |\vec{v}|^2 (1 - \cos^2 \theta) \\ &= |\vec{u}|^2 |\vec{v}|^2 \sin^2 \theta \end{aligned}$$

$$\begin{aligned}|\vec{u}\times\vec{v}|^2&=|\vec{u}|^2\,|\vec{v}|^2-(\vec{u}\cdot\vec{v})^2\\|\vec{u}\times\vec{v}|^2&=|\vec{u}|^2\,|\vec{v}|^2\sin^2\theta\end{aligned}$$

$$|\vec{u}\times\vec{v}|^2=|\vec{u}|^2\,|\vec{v}|^2-(\vec{u}\cdot\vec{v})^2$$

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$$0 \leq \theta \leq \pi$$

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