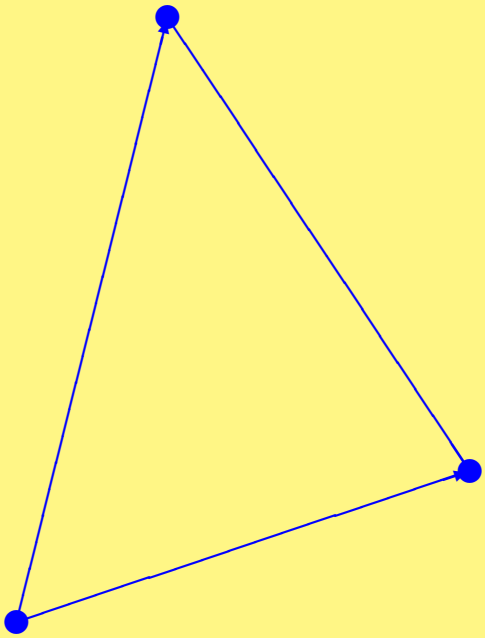
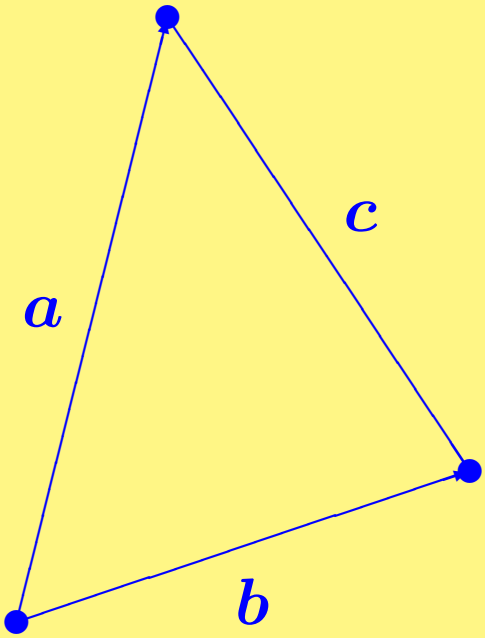
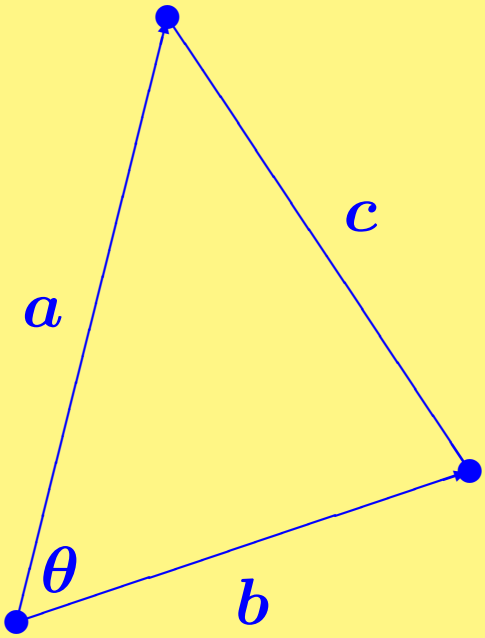
Dot Product Formula

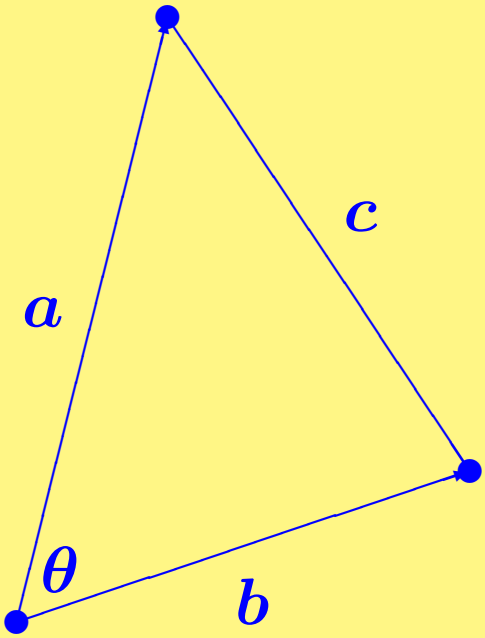
Dot Product Formula

$$\boxed{\vec{u} \cdot \vec{v} = |\vec{u}|\,|\vec{v}| \cos\theta}$$

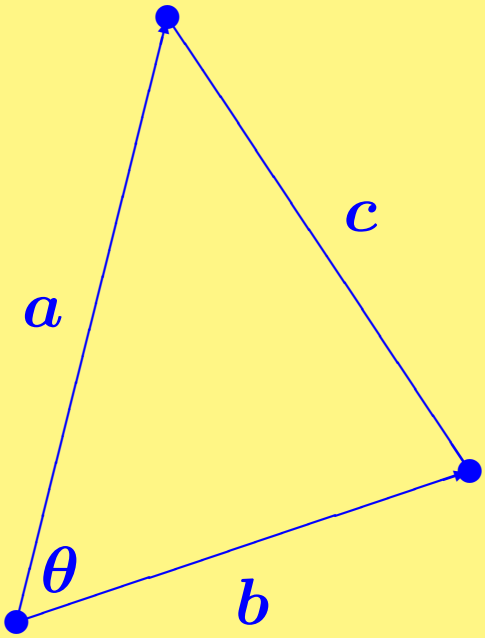



a
c
b


a
c
θ
b

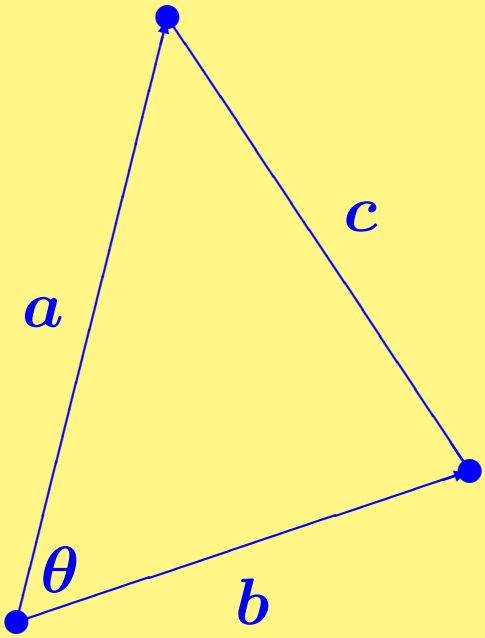


Law of Cosines:



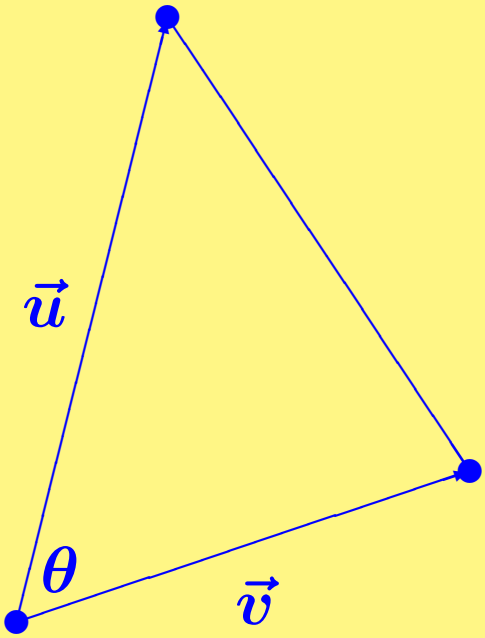
Law of Cosines:

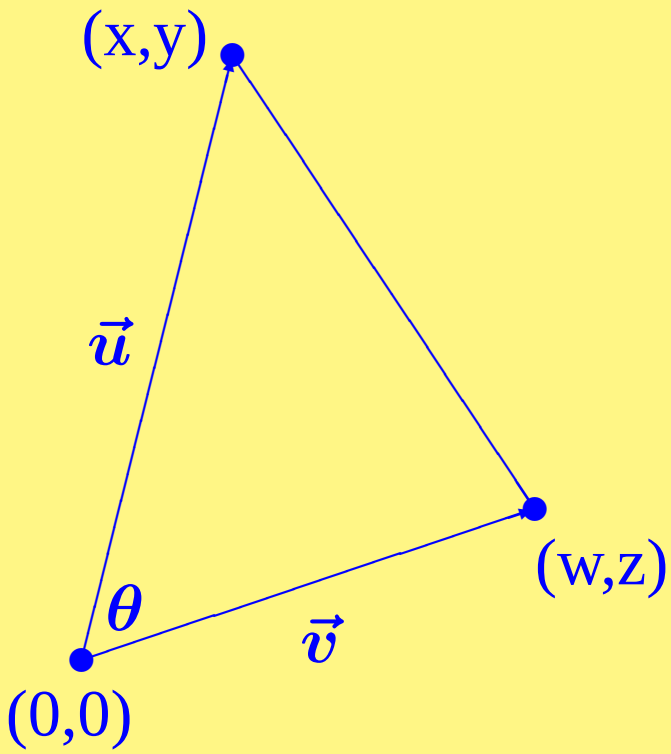
$$c^2 = a^2 + b^2$$

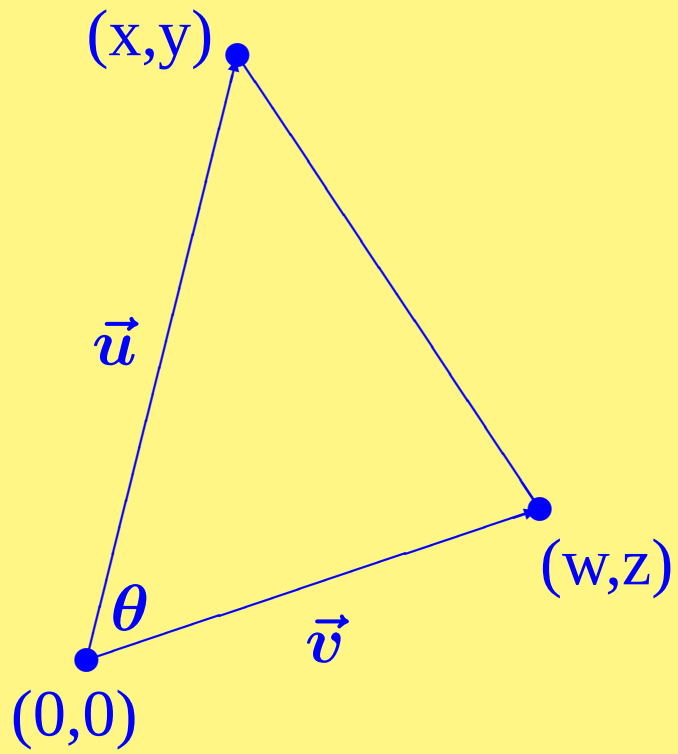


Law of Cosines:

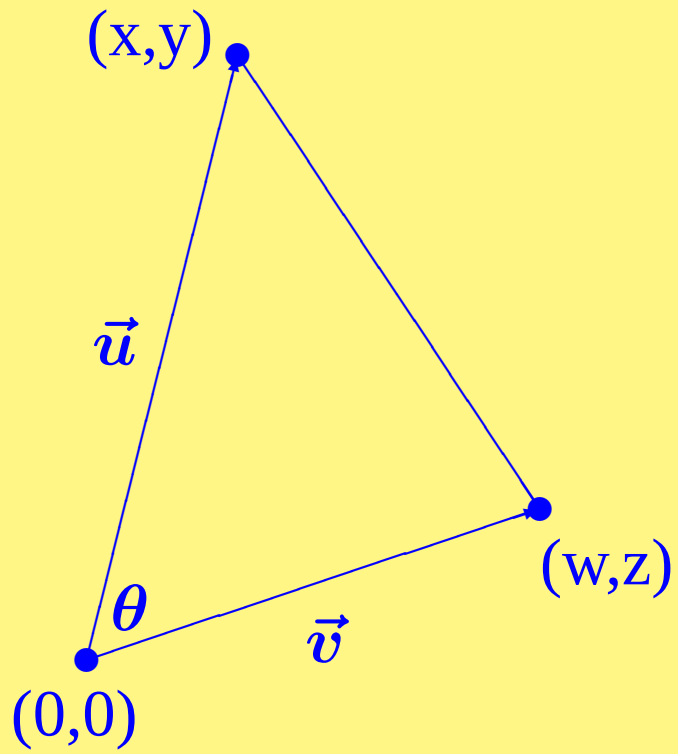
$$c^2 = a^2 + b^2 - 2ab\cos\theta$$


$\vec{u}$
θ
$\vec{v}$


(x,y)
$\vec{u}$
(w,z)
θ
$\vec{v}$
(0,0)

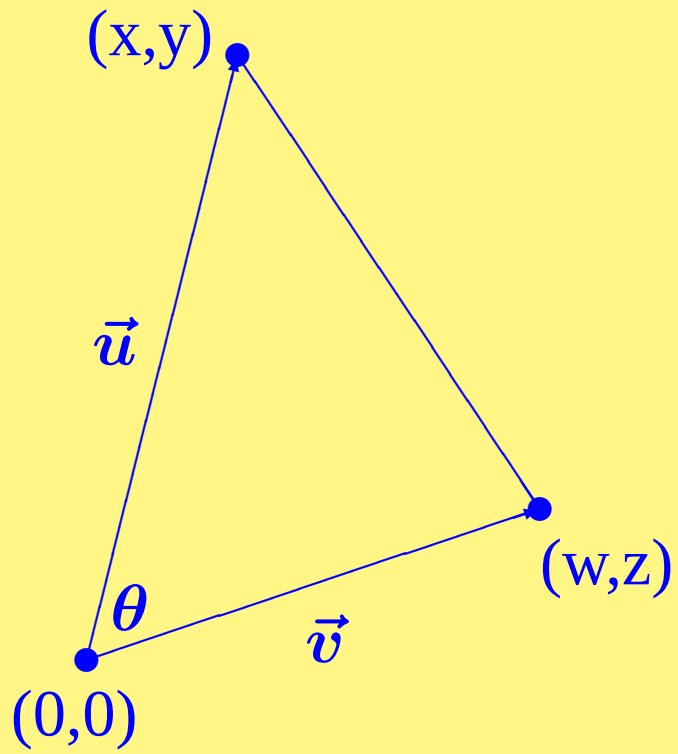


$$c^2 = a^2 + b^2 - 2ab\cos\theta$$



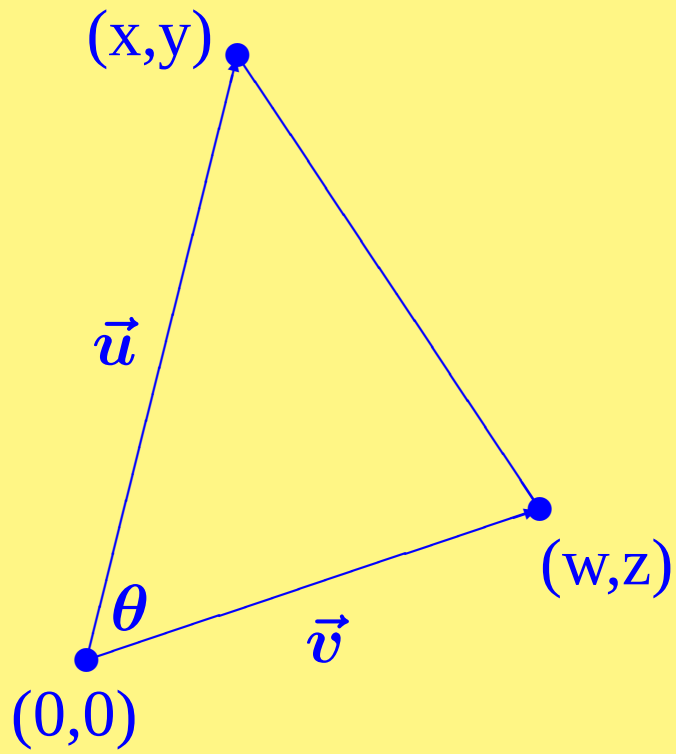
$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

$$(x-w)^2 + (y-z)^2 = a^2 + b^2 - 2ab\cos\theta$$



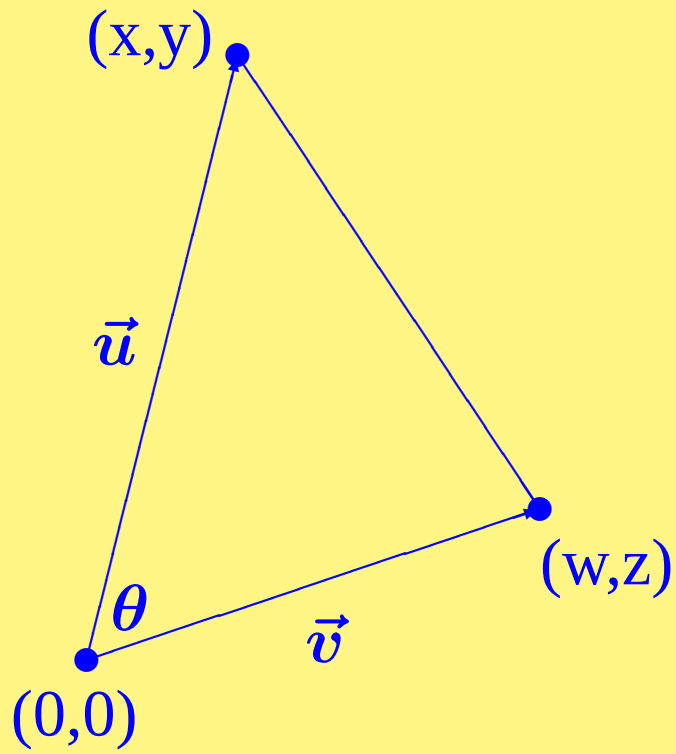
$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

$$(x-w)^2 + (y-z)^2 = |\vec{u}|^2 + b^2 - 2ab\cos\theta$$

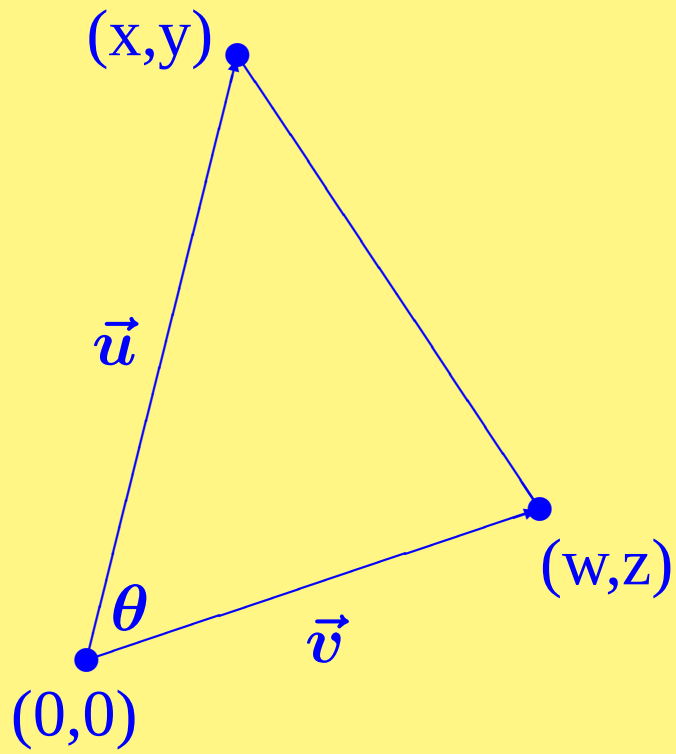


$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

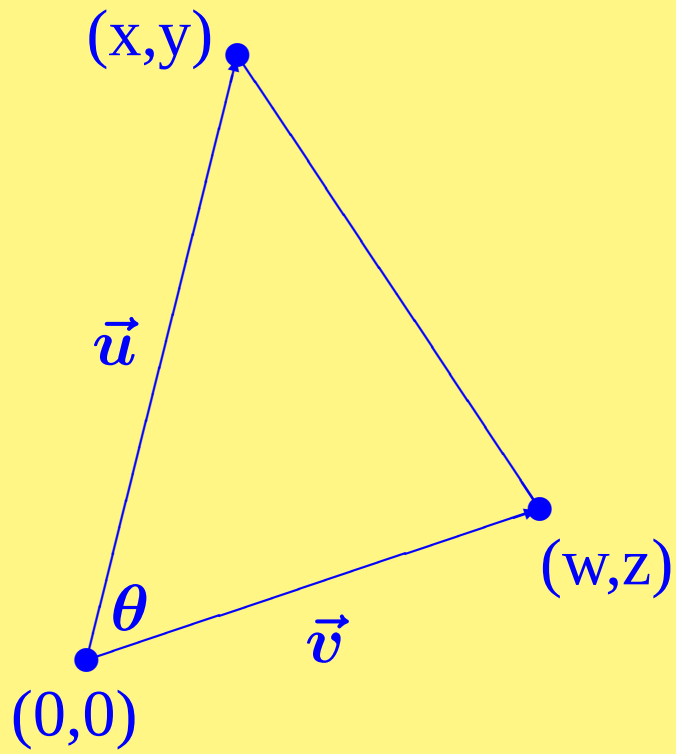
$$(x-w)^2 + (y-z)^2 = |\vec{u}|^2 + |\vec{v}|^2 - 2ab\cos\theta$$



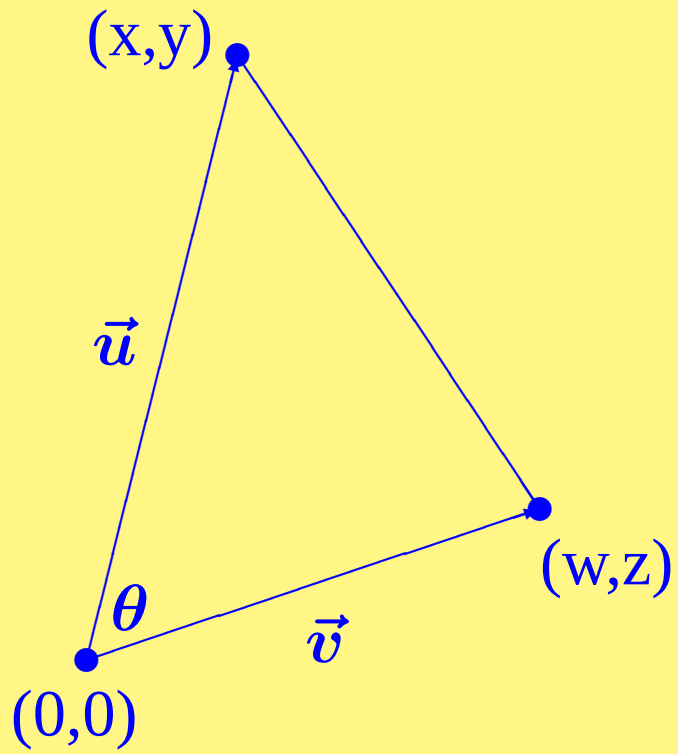
$$(x-w)^2+(y-z)^2=|\vec{u}|^2+|\vec{v}|^2-2|\vec{u}|\,|\vec{v}|\cos\theta$$



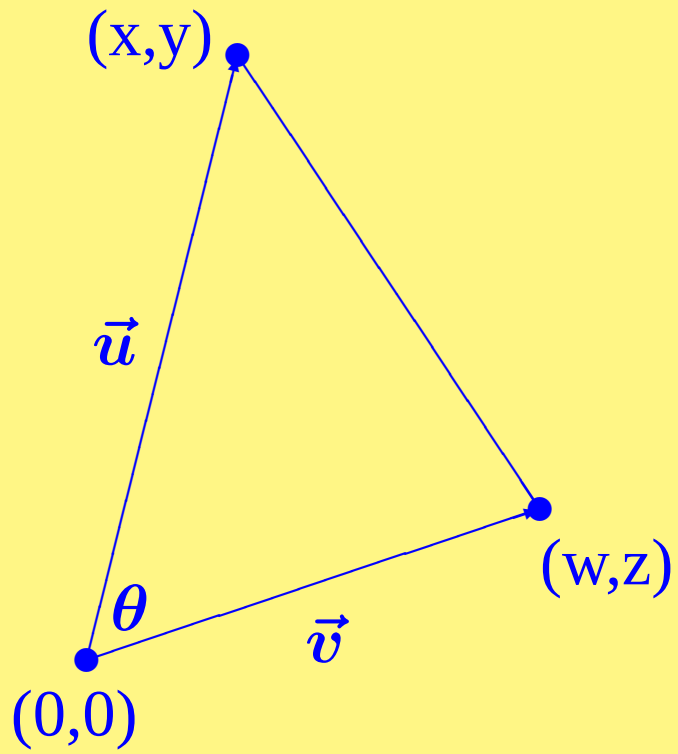
$$x^2 - 2xw + w^2 + y^2 - 2yz + z^2$$
$$= |\vec{u}|^2 + |\vec{v}|^2 - 2|\vec{u}|\,|\vec{v}| \cos\theta$$



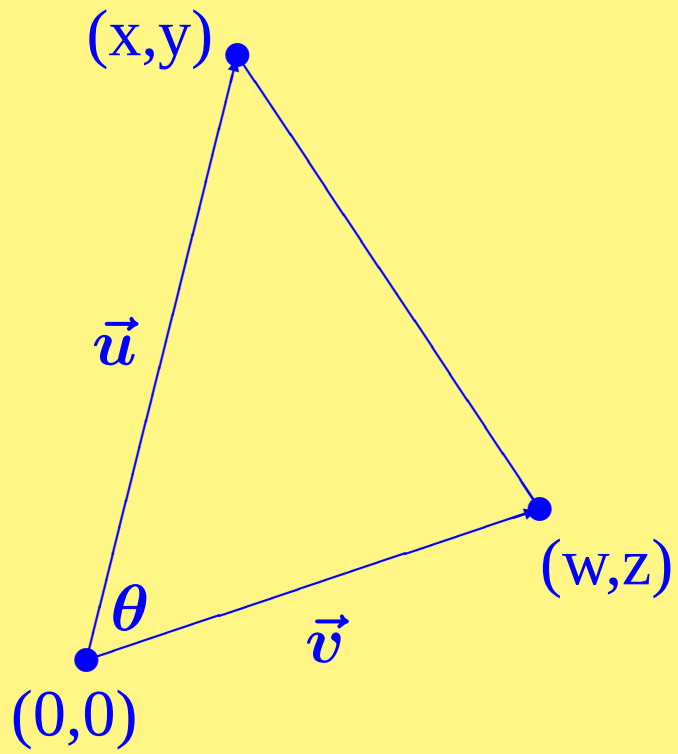
$$x^2 - 2xw + w^2 + y^2 - 2yz + z^2$$
$$= |\vec{u}|^2 + |\vec{v}|^2 - 2|\vec{u}|\,|\vec{v}| \cos\theta$$



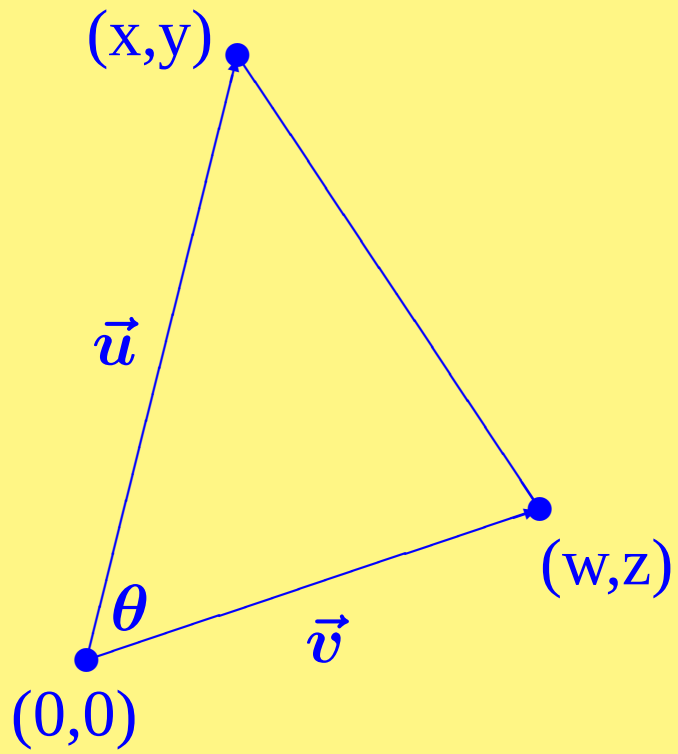
$$-2xw + w^2 + \quad -2yz + z^2$$
$$= \quad + |\vec{v}|^2 - 2|\vec{u}|\,|\vec{v}| \cos\theta$$



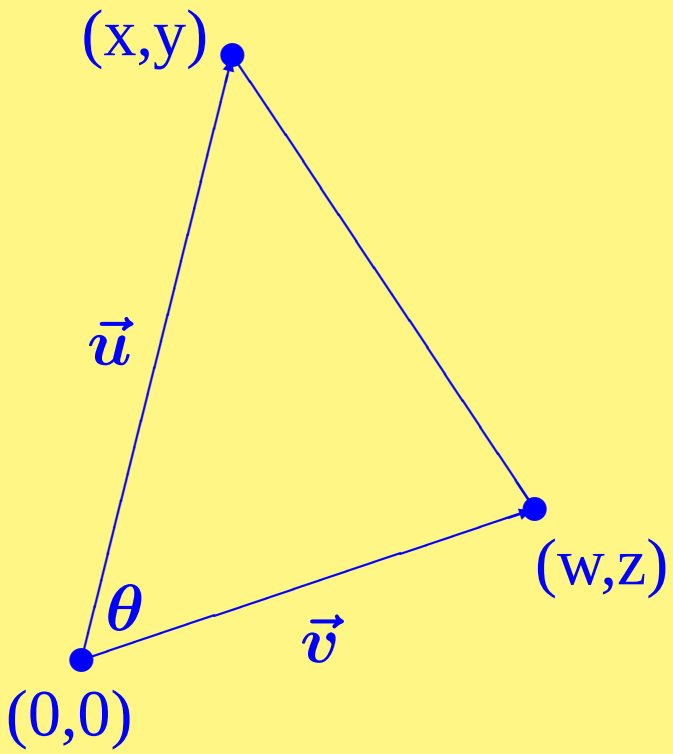
$$-2xw + w^2 + \quad -2yz + z^2$$
$$= \quad + |\vec{v}|^2 - 2|\vec{u}|\,|\vec{v}|\cos\theta$$



$$-2xw + \quad + \quad -2yz +$$
$$= \quad + \quad -2|\vec{u}|\,|\vec{v}|\cos\theta$$



$$xw + yz = |\vec{u}|\,|\vec{v}|\cos\theta$$



$$\vec{u} \cdot \vec{v} = |\vec{u}|\,|\vec{v}| \cos\theta$$