Math 241: Test 3

Name ______________________________

Instructions: Put your name in the space provided above. Make sure that your test has seven different pages. Work each problem below and show ALL of your work. Do NOT use a calculator.

Point Values: Problem (1) is worth 24 points, Problem (2) is worth 12 points, Problem (3) is worth 12 points, Problem (4) is worth 14 points, Problem (5) is worth 22 points, and Problem (6) is worth 16 points.

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(1) Calculate the following double integrals. **SIMPLIFY** your answers.

(a) $\displaystyle\int_0^1 \int_0^2 xy\, dy\, dx$

Answer: $\boxed{}$

(b) $\displaystyle\int_0^\pi \int_0^\theta r\, dr\, d\theta$

Answer: $\boxed{}$

(1) (continued)

(c) $\displaystyle\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}-x} x \cos\left(\left(\frac{\pi}{2} - y\right)^3\right) dy\, dx$ (smiplify your answer)

Answer:

(2) Calculate cylindrical coordinates (r, θ, z) and spherical coordinates (ρ, θ, ϕ) for the point with rectangular coordinates $(x, y, z) = (-\sqrt{3/2}, -1/\sqrt{2}, -\sqrt{2})$. Simplify your answers so that no trigonometric and no inverse trigonometric functions are used.

(r, θ, z): [ ]

(ρ, θ, ϕ): [ ]

(3) Define

$$f(x,y) = \begin{cases} -2 & \text{if } 0 \leq x \leq 3 \text{ and } 0 \leq y \leq 2 \\ 0 & \text{if } 0 \leq x \leq 2 \text{ and } 2 < y \leq 5 \\ 5 & \text{if } 2 < x \leq 3 \text{ and } 2 < y \leq 5. \end{cases}$$

Evaluate $\displaystyle\iint_R f(x,y)\; dA.$

Answer: [ ]

(4) Using Lagrange multipliers, calculate the distance from the origin to the nearest point on the line $x - 3y = 2$. To receive full credit for the problem, you must make appropriate use of Lagrange multipliers to derive your answer.

Distance: $\boxed{}$

(5) (a) Express the volume of the solid in the first octant and inside the cylinders $y^2+z^2=2$ and $x^2+y^2=1$ as an iterated integral in Cartesian (rectangular) coordinates. The solid is pictured on the last page of this test. Do not evaluate the integral.

Answer:

(b) For the same solid as part (a), still pictured on the last page of this test (unless you have removed the page from your test), express its volume as an iterated integral in cylindrical coordinates. Do not evaluate the integral.

Answer:

(6) Calculate

$$\int_0^4 \int_0^{\sqrt{16-z^2}} \int_{-\sqrt{16-y^2-z^2}}^{\sqrt{16-y^2-z^2}} \left(x^2+y^2+z^2\right)^{7/2}\, dx\, dy\, dz.$$

Answer: $\boxed{}$