Math 241: Test 3

Name ______________________________

Instructions and Point Values: Put your name in the space provided above. Make sure that your test has 8 different pages (including one blank page). Work each problem below and show ALL of your work. Do NOT use a calculator.

Point Values: Problems (1) through (6) are worth 12 points each. Problems (7) and (8) are worth 14 points each.

(1) Calculate the following double integrals. Simplify your answers.

(a) $\displaystyle\int_0^1 \int_x^{x^2} x\,dy\,dx$

Answer: $\boxed{}$

(b) $\displaystyle\int_0^\pi \int_0^1 dr\,d\theta$

Answer: $\boxed{}$

(2) (a) Define

$$f(x,y) = \begin{cases} 2 & \text{if } 0 \le x \le 4 \text{ and } 0 \le y \le 3 \\ 1 & \text{if } 0 \le x \le 2 \text{ and } 3 < y \le 5 \\ -3 & \text{if } 2 < x \le 4 \text{ and } 3 < y \le 5. \end{cases}$$

Evaluate $\displaystyle\iint\limits_R f(x,y)\; dA.$

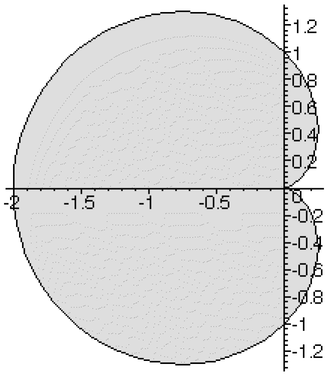
Answer: $\boxed{}$

(b) Given $f(x,y)$ in part (a), evaluate $\displaystyle\int_3^4 \int_1^5 f(x,y)\,dy\,dx.$

Answer: $\boxed{}$

(3) Calculate the area of the region inside the curve $r = 1 - \cos\theta$ (see the graph below). Simplify your answer.

Answer: $\boxed{}$



(4) Calculate rectangular coordinates (x, y, z) and cylindrical coordinates (r, θ, z) for the point with spherical coordinates $(\rho, \theta, \phi) = (6, \pi/6, \pi/6)$. Simplify your answers so that no trigonometric and no inverse trigonometric functions are used.

(x, y, z): $\boxed{}$

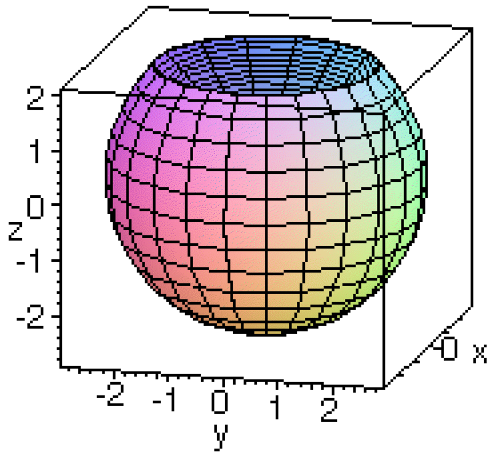
(r, θ, z): $\boxed{}$

(5) Calculate $\displaystyle\int_0^6 \int_{y/3}^2 y\sqrt{x^3+1}\ dx\, dy$. Simplify your answer.

Answer: $\boxed{}$

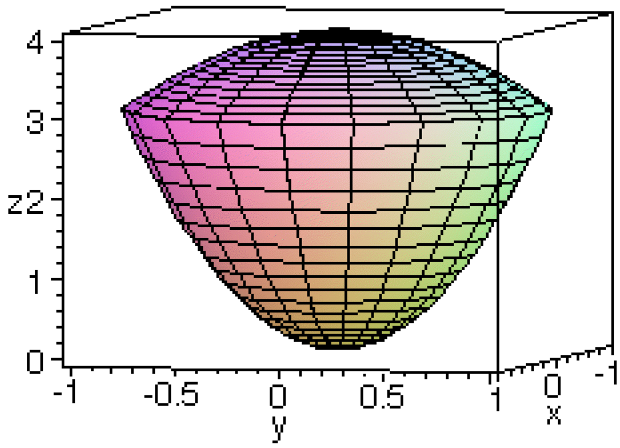
(6) Express the volume of the solid within the sphere $x^2 + y^2 + z^2 = 8$ and *outside* the half-cone $z = \sqrt{x^2 + y^2}$ as an iterated triple integral in spherical coordinates (see the graph below). Do not evaluate the integral.

Answer:



(7) Calculate the volume of the solid above the surface $z = 3x^2 + 3y^2$ and below the surface $z = 4 - x^2 - y^2$ (see the graph below). Simplify your answer. (You should use one of cylindrical or spherical coordinates; you must decide which works better.)

Answer: $\boxed{}$



(not drawn to scale)

(8) Calculate

$$\int_{-1}^{3} \int_{0}^{\sqrt{\pi}} \int_{0}^{\sqrt{\pi - y^2}} \sin\left(x^2 + y^2\right) dx\, dy\, dz.$$

Answer: $\boxed{}$