Answers to Test 1, Fall 2001

1. (a) $\langle 2, -2, 2\rangle$
   (b) $\langle 1, -8, 3\rangle$
   (c) $\langle 0, 0, 0\rangle$

2. (a) $9/2$
   (b) 8

3. (a) $z_1(x_1 + y_1) + z_2(x_2 + y_2) + z_3(x_3 + y_3)$
   (b) $z_1(x_1 + y_1) + z_2(x_2 + y_2) + z_3(x_3 + y_3)$ (possibly written differently)
   (c) From (a) and (b), it follows that $\vec{w} \cdot (\vec{u} + \vec{v}) = \vec{w} \cdot \vec{u} + \vec{w} \cdot \vec{v}$. Take $\vec{w} = \vec{u} + \vec{v}$ to get

$$(\vec{u} + \vec{v}) \cdot (\vec{u} + \vec{v}) = (\vec{u} + \vec{v}) \cdot \vec{u} + (\vec{u} + \vec{v}) \cdot \vec{v}.$$

   Taking $\vec{w} = \vec{u}$ and $\vec{w} = \vec{v}$, we obtain

$$(\vec{u} + \vec{v}) \cdot \vec{u} = \vec{u} \cdot (\vec{u} + \vec{v}) = \vec{u} \cdot \vec{u} + \vec{u} \cdot \vec{v} \quad \text{and} \quad (\vec{u} + \vec{v}) \cdot \vec{v} = \vec{v} \cdot (\vec{u} + \vec{v}) = \vec{u} \cdot \vec{v} + \vec{v} \cdot \vec{v}.$$

   Combing the equations above, we deduce

$$(\vec{u} + \vec{v}) \cdot (\vec{u} + \vec{v}) = \big(\vec{u} \cdot \vec{u} + \vec{u} \cdot \vec{v}\big) + \big(\vec{u} \cdot \vec{v} + \vec{v} \cdot \vec{v}\big) = \vec{u} \cdot \vec{u} + 2\vec{u} \cdot \vec{v} + \vec{v} \cdot \vec{v}.$$

   So (a) and (b) can be used to obtain the equation in the problem.
   (d) Use (c) to deduce that

$$\begin{aligned} 36 = |\vec{u} + \vec{v}|^2 &= (\vec{u} + \vec{v}) \cdot (\vec{u} + \vec{v}) \\ &= \vec{u} \cdot \vec{u} + 2\vec{u} \cdot \vec{v} + \vec{v} \cdot \vec{v} = |\vec{u}|^2 + 2\vec{u} \cdot \vec{v} + |\vec{v}|^2 = 16 + 2\vec{u} \cdot \vec{v} + 9. \end{aligned}$$

   It follows that

$$\vec{u} \cdot \vec{v} = \frac{36 - 16 - 9}{2} = \frac{11}{2}.$$

   From $\vec{u} \cdot \vec{v} = |\vec{u}|\,|\vec{v}|\,\cos\theta = 12\cos\theta$, we obtain that the answer is $\boxed{11/24}$.

4. (a) $\langle t^2 \cos t + 2t \sin t, t^2 \sin t - 2t \cos t, 2\rangle$
   (b) $t^2 + 2$
   (c) 15

5. (a) $2x - y = 0$ (but most wrote $4x - 2y = 0$ which is fine)
   (b) $(3/2, 1/2, -1/2)$

6. There are infinitely many correct answers here. One is $x + y = 1$.

7. (i) (d), y-axis
   (ii) (e), $z = -1$
   (iii) (a), $(0, 1, 0)$