Math 241: Quiz 9 Solutions

1. Calculate cylindrical coordinates (r, θ, z) and spherical coordinates (ρ, θ, ϕ) for the point with rectangular coordinates $(x, y, z) = (\sqrt{3}, -3, -2)$. Simplify your answers so that no trigonometric and no inverse trigonometric functions are used.

$(r, \theta, z) = \boxed{\left(2\sqrt{3}, \frac{5\pi}{3}, -2\right)}$ (simplify)

$(\rho, \theta, \phi) = \boxed{\left(4, \frac{5\pi}{3}, \frac{2\pi}{3}\right)}$ (simplify)

$$r = \sqrt{x^2 + y^2} = \sqrt{3 + 9} = \sqrt{12} = 2\sqrt{3}$$

$$x = r\cos\theta, \text{ so } \cos\theta = \frac{x}{r} = \frac{\sqrt{3}}{2\sqrt{3}} = \frac{1}{2}$$

$$y = r\sin\theta, \text{ so } \sin\theta = \frac{y}{r} = \frac{-3}{2\sqrt{3}} = \frac{-\sqrt{3}}{2}$$

$$\theta = \frac{5\pi}{3} \text{ (by previous two lines)}$$

$$\rho = \sqrt{x^2 + y^2 + z^2} = \sqrt{3 + 9 + 4} = 4$$

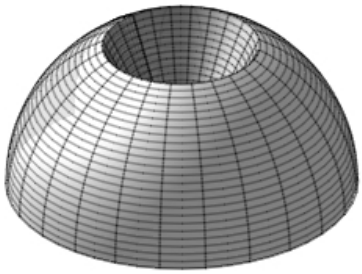
$$z = \rho\cos\phi, \text{ so } \cos\phi = \frac{z}{\rho} = \frac{-2}{4} = -\frac{1}{2} \text{ which implies } \phi = \frac{2\pi}{3}$$

2. Calculate the volume of the solid, shown to the right, consisting of all points that lie on or inside the sphere

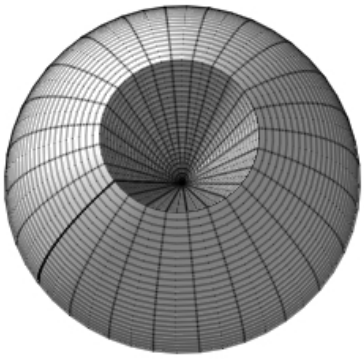
$$x^2 + y^2 + z^2 = 4,$$

on or above the xy-plane, and on or below the cone

$$z = 2\sqrt{x^2 + y^2}.$$



Simplify your answer so that no trigonometric and no inverse trigonometric functions are used.



Volume: $\boxed{\frac{32\pi}{3\sqrt{5}}}$ (simplify)

$$\int_0^{2\pi}\int_{\phi_0}^{\pi/2}\int_0^2 \rho^2\sin\phi\, d\rho\, d\phi\, d\theta$$

$$= \frac{8}{3}\int_0^{2\pi}\int_{\phi_0}^{\pi/2} \sin\phi\, d\phi\, d\theta$$

$$= \frac{8}{3}\int_0^{2\pi} (-\cos\phi)\Big|_{\phi_0}^{\pi/2} d\theta$$

$$= \frac{8}{3}\int_0^{2\pi} (\cos\phi_0)\, d\theta$$

$$= \frac{16\pi\cos\phi_0}{3} = \frac{32\pi}{3\sqrt{5}}$$

$$z = 2r \text{ so } \rho\cos\phi = 2\rho\sin\phi$$

$$\tan\phi = \frac{1}{2} \text{ (by previous line)}$$