Math 241: Quiz 9

Show ALL Work Name **Solutions**

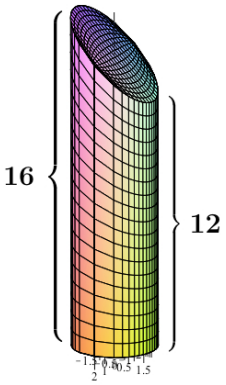
1. Recall that the volume of a cylinder of radius r and height h is $\pi r^2 h$. The double integral below represents the volume of one-half of a cylinder. Evaluate the double integral by first identifying that cylinder, by filling in the blanks below, and then calculating one-half the volume of that cylinder.

$$\iint_R (14 - y)\ dA, \qquad \text{where } R = \{(x, y) \mid x^2 + y^2 \leq 4\}.$$

The cylinder has height $\boxed{28}$ **and radius** $\boxed{2}$**. The center of the cylinder is the** $\boxed{z}$ **-axis.**

Value of Double Integral: $\boxed{56\pi}$

Solution. The bottom of the solid is a circle of radius 2 in the xy-plane, and the top is the plane $z = 14 - y$. Since $-2 \leq y \leq 2$ in the circle, the value of z varies from 12 (when $y = 2$) to 16 (when $y = -2$). The picture is at the right. Observe that if we take another copy of this picture and turn it upside down and place it on top of the original picture, we get a cylinder of radius 2 and height $16 + 12 = 28$. The double integral has value equal to one-half of the volume of this cylinder and therefore $(1/2)\pi \cdot 2^2 \cdot 28 = 56\pi$.



2. In the homework, you were asked to find the volume of the solid that is under the plane $x + 2y - z = 0$ and above the region bounded by $y = x$ and $y = x^4$. Don't do that here. Instead, simply fill in the boxes below that would make the double integral equal to this volume.

$$\int_{\boxed{0}}^{\boxed{1}} \int_{\boxed{x^4}}^{\boxed{x}} \boxed{x + 2y}\ dy\, dx$$