Math 241: Pseudo-Quiz 9

Show ALL Work Name **Solutions**

1. Determine the absolute maximum and absolute minimum value of

$$f(x, y) = x^3 - 3x + 4y^2 - 16$$

where (x, y) varies over the points satisfying $x^2 + y^2 \leq 4$. Furthermore, indicate ALL points (x, y) satisfying $x^2 + y^2 \leq 4$ where these values occur.

Absolute Maximum Value: $14/27$ ⟵ **This value occurs at:** $(-1/3, \pm\sqrt{35}/3)$

Absolute Minimum Value: -18 ⟵ **This value occurs at:** $(1, 0)$, $(-2, 0)$

Solution. For (x, y) satisfying $x^2 + y^2 < 4$, we want

$$f_x = 3x^2 - 3 = 0 \quad \text{AND} \quad f_y = 8y = 0.$$

Therefore, we want $x = \pm 1$ and $y = 0$. We have then the two points $(-1, 0)$ and $(1, 0)$ to consider here.

For (x, y) satisfying $x^2 + y^2 = 4$, we have $y^2 = 4 - x^2$ and $-2 \leq x \leq 2$, so $f(x, y) = g(x)$ where

$$g(x) = x^3 - 3x + 4(4 - x^2) - 16 = x^3 - 4x^2 - 3x.$$

We want to maximize and minimize $g(x)$ where $-2 \leq x \leq 2$. Observe that $g'(x) = 3x^2 - 8x - 3 = (3x + 1)(x - 3) = 0$, and $-1/3 \in [-2, 2]$ and $3 \notin [-2, 2]$. So we only need to consider the points where $x = -1/3$ and where $x = \pm 2$ (the endpoints of the interval $[-2, 2]$). We get the table below.

Location	x	y	Crit. Pts.	Value of f	Conclusion
inside	-1	0	$(-1, 0)$	-14	nothing worth noting
inside	1	0	$(1, 0)$	-18	abs. min.
boundary	$-1/3$	$\pm\sqrt{35}/3$	$(-1/3, \pm\sqrt{35}/3)$	$14/27$	abs. max. (on boundary)
boundary	-2	0	$(-2, 0)$	-18	abs. min. (on boundary)
boundary	2	0	$(2, 0)$	-14	nothing worth noting

(see next page for rest of quiz)

2. For this problem, you should only have to use the work you already did above. Determine the absolute maximum value and the absolute minimum value of

$$f(x, y) = x^3 - 3x + 4y^2 - 16$$

where (x, y) varies over the points satisfying $x^2 + y^2 = 4$. Furthermore, indicate ALL points (x, y) satisfying $x^2 + y^2 = 4$ where these values occur.

Absolute Maximum Value:	$\mathbf{14/27}$	$\longleftarrow$	**This value occurs at:**	$\mathbf{(-1/3, \pm\sqrt{35}/3)}$
Absolute Minimum Value:	$\mathbf{-18}$	$\longleftarrow$	**This value occurs at:**	$\mathbf{(-2, 0)}$