

Math 241: Quiz 8

Show ALL Work

Name _____

Solutions

1. Let

$$f(x, y) = x^3 + 2xy^2 - 5x^2 \quad \text{and} \quad D = \{(x, y) : x^2 + y^2 \leq 4\}.$$

(a) Find all points (x, y) where $f_x = 0$ and $f_y = 0$. You should get two points. One will be inside D , and one will be outside D .

Two Points: and

Solution: Since $f_y = 4xy$, we see that $f_y = 0$ implies that either $x = 0$ or $y = 0$. In either case, we also want $f_x = 3x^2 + 2y^2 - 10x = 0$. If $x = 0$, then $3x^2 + 2y^2 - 10x = 0$ gives us that $2y^2 = 0$ so that $y = 0$. If $y = 0$, then $3x^2 + 2y^2 - 10x = 0$ gives us that $3x^2 - 10x = 0$. Since $3x^2 - 10x = x(3x - 10)$, in this case we get that $x = 0$ or $x = 10/3$. Therefore, the points where both $f_x = 0$ and $f_y = 0$ are $(0, 0)$ and $(10/3, 0)$. ■

(b) Determine the absolute maximum value and the absolute minimum value of $f(x, y)$ on the boundary of D , where $x^2 + y^2 = 4$.

Absolute Maximum Value on Boundary of D :

Absolute Minimum Value on Boundary of D :

Solution: On the boundary, $y^2 = 4 - x^2$ so

$$f(x, y) = x^3 + 2xy^2 - 5x^2 = x^3 + 2x(4 - x^2) - 5x^2 = -x^3 - 5x^2 + 8x.$$

We therefore want to find the maximum and minimum of $g(x) = -x^3 - 5x^2 + 8x$ where $-2 \leq x \leq 2$. In theory, the instructor need not say more 😊. Since $g'(x) = -3x^2 - 10x + 8 = -(3x - 2)(x + 4)$ and -4 is not in the interval $[-2, 2]$, the maximum and minimum of $g(x)$ must occur at one of -2 , $2/3$ and 2 . Since $g(-2) = -28$, $g(2/3) = 76/27$ and $g(2) = -12$, the absolute maximum on the boundary is $76/27$ and the absolute minimum is -28 . ■

(c) Determine the absolute maximum value and the absolute minimum value of $f(x, y)$ on all of D , where $x^2 + y^2 \leq 4$. Include values of $f(x, y)$ inside and on the boundary of D .

Absolute Maximum Value: **Absolute Minimum Value:**

Solution: From part (a), the only (x, y) that we need to consider inside D and not on the boundary is $(0, 0)$. Since $f(0, 0) = 0$, which is in between the answers in part (b), we get that the answers to part (c) are the same as the answers to part (b). ■