Math 241: Quiz 7

Show ALL Work Name **Solutions**

1. There are two points on the ellipsoid $x^2 + 2y^2 + 3z^2 = 24$ where the tangent plane is parallel to the plane $2x + 4y + 6z = 15$. What are those two points? Be sure to show work.

Points: $(2, 2, 2)$ and $(-2, -2, -2)$ (Give all 3 coordinates.)

Solution: The normal to the plane $2x+4y+6z = 15$ is $\overrightarrow{n} = \langle 2, 4, 6\rangle$. At a point (x, y, z) on the ellipsoid, the tangent plane has normal $\nabla F = \langle 2x, 4y, 6z\rangle$. These planes will be parallel when these two vectors are parallel, that is when one is a non-zero multiple of the other. This happens then when there is a $k \neq 0$ such that $\nabla F = k\overrightarrow{n}$. This is the same as

$$\langle 2x, 4y, 6z\rangle = k\langle 2, 4, 6\rangle = \langle 2k, 4k, 6k\rangle.$$

So we want $x = k$ (so $2x = 2k$), $y = k$ (so $4y = 4k$), and $z = k$ (so $6z = 6k$). Since (x, y, z) is on the ellipsoid $x^2 + 2y^2 + 3z^2 = 24$, we obtain

$$24 = x^2 + 2y^2 + 3z^2 = k^2 + 2k^2 + 3k^2 = 6k^2.$$

Therefore, $k^2 = 4$ and $k = \pm 2$. Since we are interested in points where $x = y = z = k$, we get the two points listed in the answers above. ■

2. Let

$$z = x^2y - y^4, \quad x = s^2t^3 - 3st + \sin(s^2 - 1) \quad \text{and} \quad y = 3t^3 - 2s + \cos(5t).$$

Calculate $\dfrac{\partial z}{\partial s}$ at the point where $s = 1$ and $t = 0$. Simplify your answer.

Answer: -8 (Simplify.)

Solution: Given the formulas for x and y above, we see that when $s = 1$ and $t = 0$, we have $x = 0$ and $y = -1$. The chain rule gives that $\dfrac{\partial z}{\partial s} = \dfrac{\partial z}{\partial x} \cdot \dfrac{\partial x}{\partial s} + \dfrac{\partial z}{\partial y} \cdot \dfrac{\partial y}{\partial s}$. Also, at $s = 1$ and $t = 0$ and, hence, $x = 0$ and $y = -1$, we have $\partial z/\partial x = 2xy = 0$, $\partial z/\partial y = x^2 - 4y^3 = 4$ and $\partial y/\partial s = -2$. Hence, at $s = 1$ and $t = 0$, we obtain

$$\frac{\partial z}{\partial s} = 0 \cdot \frac{\partial x}{\partial s} + 4 \cdot (-2) = -8. \ ■$$