Math 241: Quiz 6   Spring, 2020   SOLUTIONS

Calculate the arc length of the curve traced by

$$x = t^3 - 3t, \qquad y = \frac{4t^3}{3} + 4t, \quad z = 3t^2$$

where $0 \leq t \leq 3$. Simplify your answer.

Arc Length: $\boxed{\quad 60 \quad}$ (Simplify)

Solution: First, we calculate

$$x'(t) = 3t^2 - 3, \qquad y'(t) = 4t^2 + 4, \qquad \text{and} \qquad z'(t) = 6t.$$

Next, we calculate

$$
\begin{aligned}
x'(t)^2 + y'(t)^2 + z'(t)^2 &= (3t^2 - 3)^2 + (4t^2 + 4)^2 + (6t)^2 \\
&= (9t^4 - 18t^2 + 9) + (16t^4 + 32t^2 + 16) + 36t^2 \\
&= 25t^4 + 50t^2 + 25 \\
&= \left(5(t^2 + 1)\right)^2.
\end{aligned}
$$

The arc length is given by

$$\int_0^3 \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2}\, dt = \int_0^3 5(t^2 + 1)\, dt = 5\left(\frac{t^3}{3} + t\right)_0^3 = 5(9 + 3) = 60. \ \blacksquare$$