Math 241: Quiz 5 Spring, 2020 SOLUTIONS

For all three problems below, A is the point $(1, -3, 2)$ and ℓ is the line given by

$$x = 1 + 2t$$
$$y = 2 + 3t$$
$$z = 1 - 2t.$$

1. Find an equation for the plane $\mathcal{P}$ satisfying the point A is on the plane $\mathcal{P}$ and the line ℓ is perpendicular to the plane $\mathcal{P}$.

   Equation of plane $\mathcal{P}$: $\boxed{2x + 3y - 2z = -11}$

   Solution: Since the line ℓ is perpendicular to the plane $\mathcal{P}$, a vector going in the direction of ℓ will be a normal to $\mathcal{P}$. So we can use $\overrightarrow{n} = \langle 2, 3, -2 \rangle$ for a normal to the plane $\mathcal{P}$. Therefore, an equation for $\mathcal{P}$ is $2x + 3y - 2z = d$ for some number d. Plugging in the point $A = (1, -3, 2)$, we see that $d = -11$. ■

2. The line ℓ and the plane $\mathcal{P}$ (in the previous problem) intersect at some point B. Calculate B.

   The point B: $\boxed{(-1, -1, 3)}$

   Solution: Plugging in the parametric equations for ℓ into the plane equation (the answer) in the previous problem, we get

   $$2(1 + 2t) + 3(2 + 3t) - 2(1 - 2t) = -11.$$

   Simplifying we obtain $6 + 17t = -11$ or $t = -1$. Plugging this value of t into the parametric equations for ℓ, we see that the point $(-1, -1, 3)$ is the intersection of ℓ and $\mathcal{P}$. ■

3. What is the shortest distance from the point A to the line ℓ? We have done this type of question in different ways. If you did the first two problems correctly, though, you should be able to use what you found above to answer this problem quickly.

   Distance from A to ℓ: $\boxed{3}$

   Solution: Since B is the point on the line ℓ that is closest to the point A, we simply compute the distance from A to B which is

   $$\sqrt{(1 + 1)^2 + (-3 + 1)^2 + (2 - 3)^2} = \sqrt{9} = 3. \blacksquare$$