Math 241: Quiz 5 Fall, 2019 SOLUTIONS

1. Find parametric equations for the tangent line to the curve given by

$$x = 1 - t^2, \quad y = 1 - t^3, \quad z = e^{t+1}$$

at the point $(0, 2, 1)$.

Parametric Equations of Line: $\boxed{\begin{aligned} x &= 2t \\ y &= 2 - 3t \\ z &= 1 + t \end{aligned}}$

Solution: Let $\vec{r}(t) = \langle 1 - t^2, 1 - t^3, e^{t+1} \rangle$. We first want to know the value of t for which $\vec{r}(t) = \langle 0, 2, 1 \rangle$. Checking, $\vec{r}(t) = \langle 0, 2, 1 \rangle$ at $t = -1$ (and no other t). Next, we compute $\vec{r}'(t) = \langle -2t, -3t^2, e^{t+1} \rangle$ and $\vec{r}'(-1) = \langle 2, -3, 1 \rangle$. The vector $\vec{r}'(-1) = \langle 2, -3, 1 \rangle$ is tangent to the curve traced by $\vec{r}(t)$ at $t = -1$, so the vector $\langle 2, -3, 1 \rangle$ is the direction of the line we want and the point $(0, 2, 1)$ is a point on the line. Hence, we get the answer above. ■

2. Find the arc length of the curve traced by

$$x = 7t - \cos t, \quad y = t + 7\cos t, \quad z = 5\sqrt{2}\,\sin t,$$

where $0 \le t \le \pi$. Simplify your answer.

Arc lenth: $\boxed{10\pi}$

Solution: Take $\vec{r}(t) = \langle 7t - \cos t, t + 7\cos t, 5\sqrt{2}\,\sin t \rangle$. The formula we want to use is

$$\text{Arc length} = \int_0^\pi \|\vec{r}'(t)\|\, dt.$$

Here, $\vec{r}'(t) = \langle 7 + \sin t, 1 - 7\sin t, 5\sqrt{2}\,\cos t \rangle$ so that

$$\begin{aligned} \|\vec{r}'(t)\|^2 &= (7 + \sin t)^2 + (1 - 7\sin t)^2 + (5\sqrt{2}\,\cos t)^2 \\ &= 49 + 14\sin t + \sin^2 t + 1 - 14\sin t + 49\sin^2 t + 50\cos^2 t \\ &= 50 + 50(\sin^2 t + \cos^2 t) = 100. \end{aligned}$$

Therefore, the answer is given by

$$\int_0^\pi \|\vec{r}'(t)\|\, dt = \int_0^\pi \sqrt{100}\, dt = \int_0^\pi 10\, dt = 10\pi. \ ■$$