# Math 241: Quiz 5

**Show ALL Work** Name **Solutions**

1. Two particles travel along the curves given by

$$\overrightarrow{r_1}(t) = \langle\, t^2,\, t,\, t^2 + t\,\rangle \quad \text{and} \quad \overrightarrow{r_2}(t) = \langle\, t^2 - t,\, 2t - 2,\, t^2 + t - 2\,\rangle.$$

There are two points where their "paths" intersect. What are they?

Two points where paths intersect: $\boxed{\begin{array}{c}(0,0,0)\\(4/9,2/3,10/9)\end{array}}$

**Replacing the variable $t$ with $s$ in $\overrightarrow{r_2}(t)$, we want to find the $t$ or the $s$ for which**

$$t^2 = s^2 - s, \quad t = 2s - 2, \quad \text{and} \quad t^2 + t = s^2 + s - 2.$$

**Plugging the second of these equations into the first gives $(2s-2)^2 = s^2 - s$. This is a quadratic equation. Squaring (and note $(2s-2)^2 = 4s^2 - 8s + 4$) and rearranging, we get $3s^2 - 7s + 4 = 0$. The quadratic formula or observing $3s^2 - 7s + 4 = (s-1)(3s-4)$ gives that $s = 1$ or $s = 4/3$. Plugging these values of $s$ in to compute $\overrightarrow{r_2}(s)$ gives that the point of intersections are $(0,0,0)$ and $(4/9, 2/3, 10/9)$. (Note: This does not verify that the curves intersect at two points. To do that one should compute $t = 2s - 2$ which gives $t = 0$ or $t = 2/3$, and check that $\overrightarrow{r_1}(0) = \overrightarrow{r_2}(1)$ and $\overrightarrow{r_1}(2/3) = \overrightarrow{r_2}(4/3)$.)** ■

2. Calculate the length of the curve given by

$$x = \frac{t^3}{3} - t, \quad y = -t^2 + 2, \quad z = \frac{t^3}{3} + t, \quad \text{where } 0 \le t \le 1.$$

Length of curve: $\boxed{4\sqrt{2}/3}$

**Taking $\overrightarrow{r}(t) = \langle x, y, z\rangle$ with $x$, $y$ and $z$ functions of $t$ as in the problem, the length of the curve is $\int_0^1 \|\overrightarrow{r'}(t)\|\, dt$. Since $\overrightarrow{r'}(t) = \langle t^2 - 1, -2t, t^2 + 1\rangle$, we have**

$$\begin{aligned}\|\overrightarrow{r'}(t)\|^2 &= (t^2-1)^2 + (-2t)^2 + (t^2+1)^2\\ &= t^4 - 2t^2 + 1 + 4t^4 + t^4 + 2t^2 + 1\\ &= 2(t^4 + 2t^2 + 1) = 2(t^2+1)^2.\end{aligned}$$

**Therefore, the length is $\displaystyle\int_0^1 \sqrt{2}\,(t^2+1)\, dt = \sqrt{2}\left(\frac{t^3}{3} + t\right)\Bigg|_0^1 = \frac{4\sqrt{2}}{3}$.** ■