Math 241: Quiz 4 Spring, 2020 SOLUTIONS

1. Find an equation of the plane containing the line $x = 1 + 2t, y = -1 + 3t, z = 4 + t$ and the point $(2, -1, 4)$.

   Equation of Plane: $\boxed{y - 3z = -13}$

   Solution[*]**:** You could do this problem by taking two different values of t to get 2 points on the line. Then those two points and $(2, -1, 4)$ give you 3 points on the plane. So if you know how to find the equation of a plane given 3 points on the plane, this can be done the same way. For this problem, we only need one point on the line, say $A = (1, -1, 4)$ (when $t = 0$). Let $B = (2, -1, 4)$ (the point given in the problem). Then A and B are on the plane, so $\overrightarrow{AB} = \langle 1, 0, 0\rangle$ is parallel to the plane. So is the vector $\overrightarrow{v} = \langle 2, 3, 1\rangle$ going in the direction of the line. So $\overrightarrow{v} \times \overrightarrow{AB}$ is perpendicular to the plane (or normal to the plane). Therefore, we can use

   $$\overrightarrow{n} = \langle 2, 3, 1\rangle \times \langle 1, 0, 0\rangle = \begin{vmatrix} \overrightarrow{i} & \overrightarrow{j} & \overrightarrow{k} \\ 2 & 3 & 1 \\ 1 & 0 & 0 \end{vmatrix} = \langle 0, 1, -3\rangle$$

   for the normal to the plane. Therefore, an equation for the plane is $y - 3z = d$ for some d. We determine d by plugging in $B = (2, -1, 4)$ or $A = (1, -1, 4)$ to get the answer. ■

2. The points that are equidistant from $(-1, 3, 2)$ and $(3, 1, 4)$ form a plane. Calculate an equation for this plane.

   Equation of Plane: $\boxed{4x - 2y + 2z = 6}$

   Solution[**]**:** Let $A = (-1, 3, 2)$ and $B = (3, 1, 4)$. We want a plane passing through the midpoint of segment $\overline{AB}$ and perpendicular to the segment $\overline{AB}$. The vector $\overrightarrow{AB} = \langle 4, -2, 2\rangle$ is perpendicular to this plane. Since the midpoint of segment $\overline{AB}$ is $(1, 2, 3)$ (which is on the plane), an equation for the plane is $4x - 2y + 2z = 6$ (or, equivalently, $2x - y + z = 3$). ■

[*]The first problem is directly from the homework (with no number changes).

[**]The second problem is the same as a homework problem with only one number changed.