Math 241: Quiz 4 Fall, 2019 SOLUTIONS

1. The plane $\mathcal{P}$ given by $7x - 4y + 4z = -3$ and the line ℓ given by $x = t + 3$, $y = 2t + 10$ and $z = -t - 1$ intersect at a point A. Calculate the point A.

   Point of Intersection A: $\boxed{(-1, 2, 3)}$

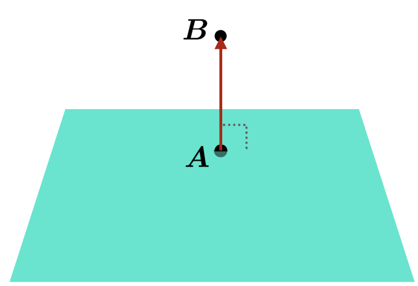
   Solution: The line is given by the points $(t + 3, 2t + 10, -t - 1)$ where t varies over all real numbers. We want to know which of these points satisfies the equation of the plane $7x - 4y + 4z = -3$. To determine this, we simply plug in the point into the plane equation and solve for t. So we start with

   $$7(t + 3) - 4(2t + 10) + 4(-t - 1) = -3,$$

   and simplify the left-hand side to get $-5t - 23 = -3$. Solving for t, we see that $t = -4$ when the point $(t+3, 2t+10, -t-1)$ in on the plane. So $A = (-4+3, 2(-4)+10, -(-4)-1) = (-1, 2, 3)$. ■

2. Let $\mathcal{P}$ and A be as in the previous problem. Calculate a point B which is a distance 18 from A and a distance 18 from plane $\mathcal{P}$. (Note: There are two correct answers. You only need to give one. Also, if you are using line ℓ, you are likely doing something wrong.)

   Point of Intersection B: $\boxed{(13, -6, 11) \text{ or } (-15, 10, -5)}$ (either answer)

   Solution: Part of the problem is to see if you can visualize correctly where B is at. The point B is directly above (or below) A so that $\overrightarrow{AB}$ is perpendicular to the plane $\mathcal{P}$ and $\|\overrightarrow{AB}\| = 18$. Thus, we are looking for B so that $\overrightarrow{AB}$ is a multiple of the vector $\overrightarrow{n} = \langle 7, -4, 4\rangle$, the normal to the plane $\mathcal{P}$ obtained from the equation for $\mathcal{P}$. Since $\|\overrightarrow{n}\| = \sqrt{49 + 16 + 16} = \sqrt{81} = 9$, and we want $\overrightarrow{AB}$ to have length 18, we can take $\overrightarrow{AB} = 2\,\overrightarrow{n} = \langle 14, -8, 8\rangle$. (Alternatively, one can compute $\overrightarrow{AB} = 18\,\overrightarrow{n}/\|\overrightarrow{n}\|$ and get the same result.) Since $A = (-1, 2, 3)$ and $\overrightarrow{AB} = \langle 14, -8, 8\rangle$, we deduce that $B = (13, -6, 11)$. The other answer is obtained from taking $\overrightarrow{AB}$ in the opposite direction, so $\overrightarrow{AB} = \langle -14, 8, -8\rangle$. Then $A = (-1, 2, 3)$ and $\overrightarrow{AB} = \langle -14, 8, -8\rangle$ gives $B = (-15, 10, -5)$.