Math 241: Quiz 3 Spring, 2020 SOLUTIONS

1. Find parametric equations for the line ℓ parallel to the line given by

$$x = -1 + 2t, \quad y = 2 - t, \quad z = 1 + t$$

and passing through the point $(-2, 2, 3)$.

Parametric Equations for ℓ: $\boxed{\begin{aligned} x &= -2 + 2t \\ y &= 2 - t \\ z &= 3 + t \end{aligned}}$

Solution*: Since the given line is parallel to (going in the direction of) the vector $\overrightarrow{v} = \langle 2, -1, 1 \rangle$, the line ℓ must be as well. Also, ℓ passes through the point $(-2, 2, 3)$. The answer follows. ■

2. Let ℓ be the line given by the parametric equations $x = 3t$, $y = 1 - 2t$ and $z = 1 + 2t$. Let ℓ' be the line given by the parametric equations $x = 2 - t$, $y = 2 + t$ and $z = -t$. The lines ℓ and ℓ' intersect at a point P. Calculate the point P. Show work and simplify your answer.

$P = \boxed{(9, -5, 7)}$ (simplify)

Solution*: Since the common point P can occur for different values of t (one for ℓ and a different one for ℓ'), we change the names of the parameters so that they are not the same and replace t in the line ℓ' with s. Then, setting the different coordinates equal, we want to know for what t and s we have

$$\begin{aligned} 3t &= 2 - s \\ 1 - 2t &= 2 + s \\ 1 + 2t &= -s. \end{aligned}$$

Adding the first two of these equations, we see that $1 + t = 4$ or $t = 3$. Plugging in $t = 3$ into the first equation, we obtain $9 = 2 - s$ so that $s = -7$. We only need to know t if we know the two lines intersect at a point, but the value of s helps us check our answer. Plugging in $t = 3$ into the equations for ℓ gives $P = (9, -5, 7)$. Plugging in $s = -7$ into the equations for ℓ' (with t replaced by s) also gives $P = (9, -5, 7)$. Thus, the two lines ℓ and ℓ' intersect at $(9, -5, 7)$. ■

*These problems were taken directly from the homework (with no number changes) that you did at home and we did in class. You're welcome.