Math 241: Quiz 2 Spring, 2020 SOLUTIONS

1. What is the volume of the parallelepiped which has vectors $\langle 1, 0, 1\rangle$, $\langle 1, 1, 0\rangle$ and $\langle 0, 1, -2\rangle$ as adjacent edges? Show work and simplify your answer.

Volume: 1 (simplify)

Solution: The volume is the absolute value of the determinant

$$\begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & -2 \end{vmatrix} = 1\,(-2-0) - 0\,(-2-0) + 1\,(1-0) = -2 + 1 = -1,$$

so the answer is 1. (Comment: The rows of the determinant come from the components of the three vectors.) ■

2. What is the height of the parallepiped in the problem above where the base is determined by the edges formed from $\langle 1, 1, 0\rangle$ and $\langle 0, 1, -2\rangle$? Show work and simplify your answer.

Height: $1/3$ (simplify)

Solution: The area of the base of the parallelepiped is the area of the parallelogram with edges $\langle 1, 1, 0\rangle$ and $\langle 0, 1, -2\rangle$. If this area is A, then we have

$$A = \|\langle 1, 1, 0\rangle \times \langle 0, 1, -2\rangle\|.$$

Since

$$\langle 1, 1, 0\rangle \times \langle 0, 1, -2\rangle = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 0 \\ 0 & 1 & -2 \end{vmatrix} = \langle -2, 2, 1\rangle,$$

we get $A = \sqrt{4+4+1} = 3$. If V is the volume of the parallelepiped and h is the height, then $V = h\,A$. From the first problem, $V = 1$. Since $A = 3$, we get $h = V/A = 1/3$. ■